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A NEW $f/0.36$ OBJECT-GLASS FOR STELLAR SPECTROSCOPY

R. J. BRACEY

ABSTRACT

A great increase in the speed of spectrographic camera lenses was obtained by Dr. W. B. Rayton, of the Bausch and Lomb Optical Company, who enlarged eightfold and suitably modified the 16-mm and the 4-mm microscope objectives and thus produced lenses of aperture ratios $f/2$ and $f/0.6$ with a useful focal length for astronomical work.

The present paper gives an account of a similar modification of the 2-mm immersion miscroscope objective which yields a lens having the aperture ratio of $f/0.36$.

An immersion fluid which can be placed in contact with the photographic emulsion has been found, which permits the use of ordinary photographic plates.

Previous to the designing of the $f/0.36$ object-glass which is the subject of the present article, very considerable advances had been made in the production of specially rapid object-glasses for astro-spectrographic work; in particular, two lenses due to Dr. Rayton of the Bausch and Lomb Optical Company had each, in turn, greatly extended the range within which the more distant nebulae could be investigated spectrographically. These advances were made as the result of increase of numerical aperture (aperture ratio), the lenses being designed on lines typical of the ordinary 16-mm and 4-mm microscope objectives much enlarged and, of course, suitably modified. The numerical aperture of the second Rayton lens was 0.85, which is practically the maximum aperture obtainable with a well-corrected "dry" lens.

The possibility of producing a lens superior in performance to the second Rayton lens depended obviously on the practicability of ob-

taining any marked increase of numerical aperture. To do so it would clearly be necessary to resort to an object-glass of the "immersion" type, i.e., a lens which would be brought into "optical contact" with the photographic emulsion by means of a suitable immersion fluid between the last lens of the object glass and the photographic plate. Such an arrangement would obviously enable a large increase in numerical aperture to be obtained, the theoretical limit being set by the refractive index (1.55) of the gelatin of the photographic emulsion. The practical limit is fixed more or less by the increased difficulty of properly mounting the back component as the aperture is increased, and by the need for using glasses of high transparency to ultra-violet light and free from tendency to become stained on their polished surfaces. It appeared probable, however, that a numerical aperture of 1.4 might be obtainable, and it was decided to aim at a design which would give this aperture.

In order that the difficulties associated with the use of curved plates or films might be avoided, it was considered worth while to attempt to design a lens giving a flat field over the range of spectrum which is of special interest in modern work on the distant nebulae. The attainment of this end appeared especially desirable in view of the fact that with the proposed type of lens it would be necessary to bring the lens and emulsion into optical contact by means of an immersion fluid, and in view also of the need for placing the plate at the proper focus of the lens within very small limits (see below).

Compared with the Rayton lenses, a lens of numerical aperture 1.4 (equivalent to $f/0.36$) may be considered as an enlarged microscope objective of the ordinary 2-mm immersion type. The accompanying table will enable the properties of the three lenses to be compared, it being understood that the values given for rapidity and depth of focus are purely relative, the corresponding properties of the 16-mm type being taken as unity.

Objective (Nominal Type)	Numerical Aperture	Relative Rapidity	Relative Depths of Focus
16 mm	0.28	1.00	1.00
4 mm	0.85	9.22	0.085
2 mm	1.4	25.00	0.043

It will be seen that there is a very considerable gain in rapidity by changing from the 4-mm type of objective to the 2-mm type. This gain is, however, accompanied by a corresponding loss of depth of focus, as is indicated by the figures in the fourth column of the table. With an object-glass of the 2-mm type, therefore, much greater accuracy would be required in making focusing adjustments; and, in using this lens, it would be important to select photographic plates coated with a suitably thin layer of emulsion, otherwise the sharpness of the photographic impression might be impaired, owing to spread of light through the emulsion beyond the point corresponding with the best focus.

The preliminary exploration of possible designs indicated that a lens to fulfil the requirements outlined above should be capable of being produced without great difficulty. Out of the designs considered, one was selected for detailed computation; and, by methods which are described later in this article, it was found possible to produce a design which appeared to be satisfactory in every respect.

It was foreseen that, owing to the adoption of an immersion type of objective, difficulties might arise if the immersion fluid to be used with the lens should prove to have any deleterious effect on the usual photographic emulsions. There appeared to be a reasonable probability that the immersion fluid which it was proposed to use would have no harmful effect on the emulsion; but as a precautionary measure the lens was designed so that, if necessary, its working distance could be made sufficient to allow of the plates being exposed with their glass surfaces in immersion contact with the object-glass. This would have necessitated the use of special thin plates made of optical glass ground and polished to a specified thickness, which would have been distinctly disadvantageous, although it would not have constituted an insuperable difficulty.

The need for specially prepared plates was found to be avoidable, however, as the result of experimental tests carried out on plates which were immersed for various periods in the immersion fluid. These tests showed that the plates were not perceptibly affected after having been kept in contact with the immersion fluid for several days, and that the plates could be readily freed from the immersion fluid by washing in a suitable solvent before development. The lens

was made up, therefore, to be used with ordinary photographic plates, the back lens having been made of the full thickness suitable for the lens to be used in immersion contact with the coated sides of the plates.

The immersion fluid selected is not very volatile; but in order to insure continuous optical contact between the back lens of the objective and the plate during the longest exposures likely to be required, special attention has been paid to the design of the cell carrying the back lens. It may be mentioned that, in tests carried out with the complete system after manufacture, optical contact could be maintained between the emulsion and the lens for periods exceeding ten days.

To secure the necessary accuracy of focus the plate-carrier was originally designed as an integral part of the lens mounting. Focusing adjustment is provided by means of a differential screw fitted with a clamping adjustment, so that, when once the proper focus has been found, it may be retained permanently; there should be no need ever to vary this adjustment, provided that the lens is always set up so that light of a specified color or wave-length is always directed along the axis of the lens. In addition to the focusing adjustment, the plate carrier is fitted with a tilting adjustment. This was considered desirable because, although the focal surface of the lens is always flat, it will not be quite perpendicular to the optical axis of the lens unless the lens is set up so that light of a particular color passes accurately parallel to the axis. The tilt of the focal surface depends on the wave-length of the light which is directed along the axis of the lens, and the fitting of the tilting adjustment will permit of the convenient compensation of any slight error of alignment in setting up the system. When the correct tilt of the plate-carrier has been found, the tilting adjustment may be clamped and thereafter no further adjustment will be required, since each fresh plate will register automatically in the proper focal plane.

With the original plate-holder it was necessary for the lens and plate-carrier to be removed as a whole to the darkroom when plates were to be loaded and unloaded. This design, which was used for an instrument constructed for the California Institute of Technology, was subsequently modified, during tests at the Mount Wilson Ob-

servatory, to avoid danger involved in transporting the entire apparatus between the darkroom and the observing platform. Plates can be loaded and unloaded without disturbing the lens; but the main features of the original design, such as the focusing and tilting adjustments, have been retained.

The lens was designed by R. J. Bracey, of the British Scientific Instrument Research Association, and was corrected to suit the collimator and prism system made by Messrs. Ross, Limited, London, England, with which the lens is to be used. Messrs. Ross provided full particulars of this system in order that the objective might be suitably corrected. The construction of the objective was undertaken by Messrs. R. and J. Beck, Limited, London, England; and the complete system was examined optically, after manufacture, in the laboratories of the British Scientific Instrument Research Association, and trial photographs were taken with the lens before delivery. The performance of the complete system, as revealed by this examination, indicated that the system fully realized the expectations founded on the calculations made in working out the final design.

For the examination of the system a laboratory method was developed for testing and adjusting the focusing and tilting adjustments. The same method will enable the system to be adjusted accurately before it is mounted in the telescope; photographs of stellar spectra should therefore be obtainable without there being any loss of time in making trial exposures to test the various adjustments.

Some brief account of the problems which arise in designing a lens of this type may possibly be of interest. The light which is to be analyzed into its constituent wave-lengths enters the collimator through a slit on which an image of a star or nebula is focused. The light is dispersed by the prism train after passing through the collimator object-glass, and enters the photospectrographic objective as a series of beams of parallel light of different colors traveling along directions which are variously inclined to the axis of the objective. The purpose of the objective is to form from each of these dispersed beams a real image of the slit, and these images should preferably all lie in one plane.

Such a lens may be considered as a type distinct from all the usual types of object-glasses, in the sense that the corrections required can

be obtained by methods somewhat different from those which must be used for correcting other kinds of object-glasses. For example, although chromatic aberration, spherical aberration, coma, curvature of field, and astigmatism must be taken into account in designing the lens, achromatism, as the term is applied to ordinary objectives, is not essential, and a certain amount of astigmatism may be tolerated in the images formed by the light which travels along directions inclined to the axis.

The problem of designing such an object-glass is, of course, a matter of suitably balancing and manipulating the optical constants and aberrations of the system. An object-glass entirely free from secondary spectrum, and with an anastigmatically flattened field, would be ideal for use as an astrospectrographic object-glass, provided such a lens could be made with sufficiently large aperture. An object-glass of the ordinary achromatic type corrected so as to have an anastigmatically flattened field would not, however, be of much service; its secondary spectrum would cause light of different colors to come to focus in different planes, all parallel to each other but intersecting the axis of the lens at different points. The focal plane for the "mean ray," i.e., for light of the wave-length for which the lens was originally achromatized, would be nearer the lens than the focal planes of any of the other colored beams. If such a lens, used with a collimator and prismatic system, were adjusted so that light of the color of the "mean ray" were directed parallel to the axis of the lens, all light of shorter wave-lengths would pass obliquely through the lens, crossing the axis in one direction, while all light of longer wave-lengths would pass obliquely through the lens, crossing the axis in the opposite direction. The inclination of these sets of rays would increase as the difference of wave-length from the "mean ray" increased, and the focal points of the various colored beams would all be farther from the lens than the focal point of the "mean ray"; the focal surface, i.e., the surface which passes through the various colored images, would, in fact, be a curved surface with its convex side toward the lens. An achromatic lens which gives an anastigmatically flattened field is therefore unsuitable for use as a spectrographic objective. An achromatic lens could, however, be made to give an accurately focused spectrum on a flat plate, provided that, when used

as an ordinary lens, its field were curved in such a way and to such an extent that increase in focal distance due to change of color (difference of wave-length) exactly compensated the reduction of focal distance due to field curvature.

It follows from the foregoing that, for a given chromatic dispersion by the prism system, an achromatic objective can be specified having a certain radius of curvature of field in the ordinary sense which, with the proper balance of its other aberrations, would cause the various colored images all to lie in one plane when the lens was used as a spectrographic objective.

The way in which a spectrographic objective is used and the type of definition required are such as to allow of appreciable astigmatism in the images formed by the oblique rays. The objective is used with a collimator into which the light enters through a slit; and the various colored images of the slit will all lie in the focal plane with their centers on a straight line passing through the optical axis of the lens and with their lengths perpendicular to this straight line. For identification of the lines, or for measurements of displacements of the lines, the only matter of importance is that the slit image should be sharply defined across its width, in order that the distances of the slit images from the center of the plate can be accurately determined. In correcting the lens, therefore, astigmatic correction need not be specifically considered, it being sufficient to design the lens so as to make only one of the focal surfaces (the tangential astigmatic surface) of the prescribed curvature. It happens that, in order to produce the desired curvature of the tangential focal surface, the astigmatism requires to be a little overcorrected.

It has already been mentioned that the design of the *f*/0.36 objective follows more or less closely the type of design used for microscope objectives of 2-mm focus. The field of such a lens is strongly curved, and the residual or secondary chromatic errors are small and more or less fixed in amount. The directions along which light of different colors passes through the objective are fixed by the prism system; and it was found, in developing the design, that in the angle over which the light was spread by the prism the curvature of field was greater than could be compensated by merely taking advantage of the residual chromatic aberration of the lens. It was necessary,

therefore, to supplement the "secondary spectrum correction" by flattening the tangential astigmatic field of the object-glass.

The method used to flatten the tangential astigmatic field was one which, in addition, introduced undercorrection for spherical aberration in oblique image-forming rays. If the ordinary type of object-glass has its spherical aberration removed for the "mean ray," then for light of shorter wave-length there will be overcorrection of spherical aberration. In the system under consideration, however, comprising the collimator, prisms, and the $f/0.36$ lens, the obliquity of the ray systems increases as the wave-length of the light becomes shorter; and the undercorrection at this increased obliquity due to the method of adjusting the field curvature can be made to balance the overcorrection due to the shortening of the wave-length. Hence, proceeding from the center of the field of view outward in the direction of shorter wave-lengths, correction for spherical aberration can be maintained for a long way. In the reverse direction, spherical aberration is increased more rapidly than it would be if the field were not flattened; but this is of no importance, since it occurs only in the shortened red end of the spectrum.

The object-glass, as finally designed, exhibits full spherical correction over a range of wave-lengths from 3600 A to 5000 A in addition to having a flat field; in the design, also, particular attention has been paid to insuring that the sine condition should be fulfilled and to eliminating any independent coma of a higher order, so that the entire spectrum should be free from coma.

British Scientific Instrument Research Association
London

POSTSCRIPT

The remarkable lens described above by Mr. Bracey, which was suggested in principle by Sir Herbert Jackson, was very kindly developed and tested for the California Institute of Technology by the British Scientific Instrument Research Association, now under the direction of Dr. Harry Moore. When tested recently at the Mount Wilson Observatory by Mr. Humason, it was found to be admirably adapted for the solution of certain problems of the extra-galactic nebulae on which Dr. Hubble and Mr. Humason are engaged. Its speed is so great that its only limitation is fixed by the brightness of the sky background. When used at Palomar Mountain with the 200-inch telescope, it is expected to be especially efficient.

George E. Hale

KINEMATICS AND WORLD-STRUCTURE. II

H. P. ROBERTSON

ABSTRACT

The *equations of motion* of a test particle in an idealized universe satisfying the cosmological principle are obtained from the standpoint of the general kinematics developed in Part I of this paper, and are found to be determinate only to within an arbitrary function Γ of two variables. Their integration is reduced, in principle, to the solution of a single first-order ordinary differential equation. The *acceleration function* Γ is determined and the integration completed upon imposing (*a*), the general relativistic theory of gravitation, thus showing the consistency of the general kinematics here developed with that theory, and also upon imposing (*b*), any suitable adaptation of the Newtonian theory of gravitation, illustrated in detail by a relativistic extension applicable to the case in which the total mass of the fundamental particles (nebulae) is finite.

INTRODUCTION

We continue here our investigation[1] of the idealized cosmological problem in which the nebulae are replaced by a set of fundamental particles satisfying a general uniformity postulate. It was shown in Part I that, if observers associated with these fundamental particles are to experience identical views of the universe, there necessarily exists an invariant Riemannian metric (2.1) in terms of which the given elements—clocks, theodolites, and light-signals—are interpretable in exactly the same way as in the general theory of relativity. We here extend this kinematical foundation by an investigation of the motion of a test particle in such a universe, and examine the relationship of the results obtained to certain theories of gravitation. The investigation of statistical systems satisfying the uniformity postulate and the examination of the relationship of the whole to Milne's kinematical-statistical theory of gravitation are left to the concluding part, III, of this paper.

4. EQUATIONS OF MOTION OF A TEST PARTICLE

Allow, now, each of the preferred observers the possibility of projecting at will a test particle from any event on his world-line in any timelike direction; we here assume that this may be done with but

[1] H. P. Robertson, "Kinematics and World-Structure," *Ap. J.*, **82**, 284–301, 1935; hereinafter referred to as "Part I." The numbering of sections and equations is carried over from this previous work; thus, references to secs. 1–3 and to eqs. (0.1)–(3.3) are to those of Part I.

negligible effect on the metric, and that the reaction on the observers may be ignored. Clearly, we do not thereby relinquish the uniformity postulate on which the previous analysis was based, for, since each observer has exactly the same possibility of projecting test particles as any other, these particles cannot be used to establish absolute position or orientation in the spaces $\tau=\text{const}$. But, equally clearly, the subsequent motion of such a test particle is severely restricted by the cosmological postulate, for, if A projects at time τ a test particle P under certain initial circumstances, and if at the same time any other observer A′ projects a particle P′ under precisely the same relative circumstances, the subsequent motion of P relative to A must be identical with that of P′ relative to A′. It is this restriction which we wish to examine here.

Consider, then, the motion of such a test particle P as viewed by any preferred observer A. He assigns to each event E on the world-line C of P co-ordinates $\eta^\mu=(\tau, \eta^a)$, as described in section 2 of Part I, and C is in its entirety specified by considering the η^μ as functions of some appropriate parameter. Three such parameters immediately suggest themselves: the cosmic time τ of the event, the invariant interval s measured with the aid of the metric (2.1) along C from some arbitrarily chosen origin, and the length u of its projection C* in the auxiliary u-space with metric (0.2). Of these, the most convenient for our immediate purpose is the invariant space-time interval s, the quantity which is, in the theory of relativity, the "proper time" of the particle—although we here in no way imply that it is to be measured by a clock which shares the motion of P. The tangent at E to the curve C, which may be considered as the four-dimensional analogue of velocity, is specified by the unit vector

$$\zeta^\mu \equiv \frac{d\eta^\mu}{ds}, \qquad g_{\mu\nu}\zeta^\mu\zeta^\nu = 1, \tag{4.1}$$

where the $g_{\mu\nu}(\tau, \eta^a)$ are the coefficients of the metric (2.1) and repeated indices are, in accordance with the usual conventions, to be summed over their range.

We now make explicit the assumption, tacitly adopted in the foregoing, that the equations of motion are, as in classical and relativistic dynamics, to be expressed in terms of, at most, second dif-

ferentials of the co-ordinates η^μ—i.e., that apart from singular events, such as those defined by $\xi(\tau)=0$, the circumstances of projection described above define uniquely the subsequent motion. The "acceleration" vector of P, which may without loss of generality be represented by

$$A^\mu \equiv \frac{d\zeta^\mu}{ds} + \left\{ {\mu \atop \nu\sigma} \right\} \zeta^\nu \zeta^\sigma \, , \tag{4.2}$$

must therefore be a vector function of, at most, the event $E(\eta^\mu)$ and the direction vector ζ^μ at E of C. Furthermore, the cosmological postulate requires that the equations of motion thus obtained be invariant in form, as well as in fact, under the group G_6 of transformations on the spatial co-ordinates η^μ. Now, the only two independent vectors intrinsic to the problem are the unit vector ζ^μ and δ^μ_0 (characterizing the motion of that preferred observer whose world-line passes through the event E in question); and, since but one of these has components in the space $\tau=$const. of the variables η^a, no further vector can be constructed from them by vector multiplication, even though we are only interested in transformations involving the three co-ordinates η^a. Hence the equations of motion must be of the form

$$A^\mu = \Gamma \cdot \delta^\mu_0 + \Delta \cdot \zeta^\mu \, , \tag{4.3}$$

where Γ, Δ are scalars which are invariant in form under the group G_0. The only two independent scalars satisfying this requirement are the single point-invariant τ and the scalar product $g_{\mu\nu}\zeta^\mu\delta^\mu_0 = \zeta^0$ of the two vectors, which quantity we denote for convenience simply by ζ; hence Γ and Δ can, at most, be functions of these two scalars. But Γ, Δ are not both arbitrary, for, since ζ^μ is a unit vector its contravariant rate of change A^μ is orthogonal to it in the sense of the metric, i.e., the inner product $A_\mu\zeta^\mu$ vanishes, as may be seen by differentiating the normality condition (4.1) along the curve C. Hence by (4.3) $\Gamma\cdot\zeta+\Delta\cdot 1$ must vanish, and the equations of motion assume the form

$$A^\mu = \Gamma(\tau, \zeta)[\delta^\mu_0 - \zeta\zeta^\mu] \, . \tag{4.4}$$

The second equation of (4.1) is now an integral of these equations (4.4) and may therefore replace one of them—say that given by

$\mu=0$. It is readily seen that the world-lines $\eta^a=\text{const.}$ ($\zeta=1$) of the preferred observers formally satisfy the equations of motion—as do also the world-lines of light, as shown below.

In this way the cosmological principle reduces the problem of motion to that of determining, by the imposition of physical law, a single scalar function Γ (τ, ζ) of two variables whose physical dimensions are those of $(\text{time})^{-1}$. Apparently Γ could also depend on some physical attribute of the test particle; however, the only one pertinent to the applications we have in mind is mass; and, since we shall assume the equality of inertial and gravitational mass, the acceleration, and therefore Γ, will be independent of it.

It may be objected that the use of A^μ as acceleration is illegitimate, as we have refused to identify s with any clock-time. But it is clear from general invariant-theoretical considerations that this can in no essential way alter our conclusions, and that, by starting with any other independent variable, such as τ, we must by corresponding steps arrive at equations which are identical in form with those obtained from (4.4) on transforming to the new independent variable τ. The result of performing this transformation on the three equations corresponding to $\mu=a$, which will be of use in the sequel, is found to be

$$\frac{dv^a}{d\tau}+\left\{\begin{matrix} a \\ \beta\gamma \end{matrix}\right\} v^\beta v^\gamma=-\zeta^{-2}\left(\Gamma+\frac{(1+\zeta^2)\xi'}{\xi}\right)v^a\,, \tag{4.5}$$

where v^a, ξ' are the derivatives of η^a, ξ, respectively, with respect to τ. These three equations are to be supplemented by the integral (4.1), which is here most conveniently taken in the form

$$\zeta^{-2}=1-\xi^2(\tau)v^2\,, \qquad v^2\equiv h_{a\beta}v^a v^\beta\,. \tag{4.6}$$

5. INTEGRATION OF THE EQUATIONS OF MOTION

We have already noted that (4.1) or (4.6) represents a first integral of the equations of motion (4.4). We next remark that the cosmological principle demands that the path $C^*: \eta^a=\eta^a(s)$ traced out in the auxiliary space with metric (0.2) must be a geodesic of that space, as otherwise its deviation from a tangent geodesic would determine a preferred spatial direction, and this would belie the assumed iso-

tropy.[2] There now remains only the determination of the distance u, measured by the metric (0.2) along this geodesic C* of the auxiliary three-space, as a function of s or τ, for, as shown below, the finite equations of the geodesics of (0.2) can be found directly from the finite transformations of the group G_6.

In order to determine the distance u along C* as a function of τ, we find it convenient to introduce the scalar

$$\omega \equiv \xi^2(\tau) \left|\frac{du}{ds}\right| , \qquad \left(\frac{du}{ds}\right)^2 = h_{\alpha\beta}\zeta^\alpha\zeta^\beta , \tag{5.1}$$

in place of ζ, to which it is related by the equation

$$\zeta^2 = 1 + \frac{\omega^2}{\xi^2(\tau)} , \tag{5.2}$$

obtained with the aid of the normality condition (4.1); this new quantity ω is roughly analogous to areal velocity in the classical central-force problem. We shall here, as in certain developments in the sequel, consider τ and ω as the fundamental scalars in place of τ and ζ; we shall in such cases simply write $\Gamma(\tau, \omega)$ for the acceleration function Γ when considered as a function of these variables. On differentiating the defining equation (5.1) for ω with respect to s and eliminating $d\zeta^\alpha/ds$ with the aid of (4.4), we obtain a result which may, on dividing by ζ, be expressed in the form[3]

$$\frac{d\omega}{d\tau} = -\omega\Gamma(\tau , \omega) . \tag{5.3}$$

The problem of integration is thus reduced to that of solving the first-order ordinary differential equation (5.3) for ω as a function of τ, and a subsequent quadrature. Although the actual solution of this equation must await a more precise determination of Γ, we can

[2] This result may, of course, be derived directly from the equations (4.4) on going over to the arc-length u as parameter—as was done for the case $\Gamma = 0$ in Appendix E of the author's report "Relativistic Cosmology," *Rev. Mod. Phys.*, **5**, 62–90, 1933. This derivation is instructive, as it yields as a by-product the eq. (5.3) derived below.

[3] The values of the Christoffel symbols involved in this computation are given, e.g., in Appendix A of the report referred to in n. 2 above.

nevertheless proceed formally to the solution of the kinematical problem with the aid of the formal solution

$$\omega = f(\tau, \omega_0) , \tag{5.4}$$

where $\omega_0 \geqq 0$ is the value assumed by ω at some given non-singular epoch τ_0. If in particular $\omega_0 = 0$, then initially du/ds and therefore all ζ^a vanish; in this case the geodesic C* degenerates into the point $\eta^a = \text{const.}$, and we may consequently characterize the world-lines of the preferred observers as those trajectories (4.4) for which $\omega = 0$. If, on the other hand, we allow $\omega_0 \to \infty$, we obtain null-curves $ds = 0$ whose projections C* in the auxiliary space (0.2) are geodesics, and which are therefore (as shown in Part I, p. 296) themselves null-geodesics of the space-time (2.1); all light-paths can thus be characterized as trajectories for which $\omega = \infty$.

Granting the solution of the differential equation (5.3), the integration of the equations of motion is reduced to a single quadrature. For, on solving (5.1) for $du/d\tau = du/\zeta ds$ and substituting for ζ its value (5.2), we find, on separating variables and integrating,

$$u - u_0 = \pm \int_{\tau_0}^{\tau} \frac{\omega d\tau}{\xi^2(1+\omega^2/\xi^2)^{\frac{1}{2}}} , \tag{5.5}$$

where u_0 is the value assumed by u at the epoch τ_0. The original parameter s can, if desired, be obtained as a function of τ by integrating the equation (5.2) for $\zeta = d\tau/ds$.

We return for a moment to the problem of determining the co-ordinates η^a directly as functions of τ. We first remark that the homogeneity and isotropy of the auxiliary space allows us to introduce co-ordinates $\eta^a = (\eta, \theta, \varphi)$, as defined by equation (0.3) of Part I, in such a way that the initial point u_0 of C* may be taken as the origin of η; and the polar co-ordinates θ, φ may be so chosen that at this point the direction of the curve C* lies in the intersection of the equatorial surface $\theta = \pi/2$ and the meridian surface $\varphi = 0$. But this intersection is itself a geodesic, and hence it is the curve C* which we are seeking; in this co-ordinate system the finite equations of the trajectory C in space-time are therefore given by

$$\eta^0 = \tau , \qquad \eta^1 = u , \qquad \eta^2 = \frac{\pi}{2} , \qquad \eta^3 = 0 , \tag{5.6}$$

where u is that function of τ defined by the integral (5.5). The equations of C in terms of the co-ordinates τ, η'^{a} of an arbitrary fundamental observer may then be obtained by replacing the variables η^{μ} by their values (5.6) in the equations of that transformation of G_6 which sends η^{μ} into η'^{μ}; we are here not interested in the explicit form of these results, for (5.6) will suffice for our purposes. We note in passing, however, that this general problem is closely related to that of treating the subject in terms of the theory of distant parallelism, which proves particularly significant in the case $k=+1$ in which (0.2) defines elliptic space. For, in this case, the fundamental ennuple ("4-Bein") may be transported by Clifford parallelism, and the equations of motion (4.5) assume a much simpler form on expressing the velocity v^{μ} in terms of its components with respect to the ennuple; four first-integrals are immediately available without previous specialization of the co-ordinate system, yielding the direction cosines ζ^{μ} of the trajectory directly in terms of the co-ordinates η^{μ} and three arbitrary constants, subject, of course, to the solution of (5.3).[4]

Let us attempt, without going too deeply into the matter, to gain some notion of the qualitative features of the trajectories in a broad class of models suggested by those already proposed in other connections. Consider, then, the ultimate fate of a test particle in a universe in which $\xi(\tau)$ increases eventually at least as fast as some constant multiple of τ, and in which ω is bounded as $\tau \to \infty$. Under these conditions u, defined as a function of τ by (5.5), approaches a finite limit u_{∞} as $\tau \to \infty$; furthermore, the "distance" $\xi(\tau)(u_{\infty}-u)$ of this test particle from that preferred observer characterized by $u=u_{\infty}$ in general tends toward zero, or at worst (for $\xi \sim \tau$) toward some finite limit d. But this means that each such particle eventually becomes associated with some fundamental particle A by approach-

[4] The analytical apparatus necessary for this treatment has been developed in secs. 9 and 12 of the author's "Groups of Motions in Spaces Admitting Absolute Parallelism," *Ann. Math.*, **33**, 496–520, 1932. It should be remarked that there is but little connection between this extension of Clifford parallelism to cosmological space-times and the formal absolute parallelism employed by G. C. McVittie in his interesting papers "Absolute Parallelism and Milne's Kinematical Relativity," *M.N.*, **95**, 270–279, 1935; "Absolute Parallelism and the Expanding Universe Theory," *Proc. R. Soc.*, A, **151**, 357–370, 1935; "Gravitation in Cosmological Theory," *Zs. f. Ap.*, **10**, 382–390, 1935.

ing asymptotically either the world-line of A itself or a line which parallels it at a finite distance d. If, on the other hand, we follow the trajectory back in those cases in which there is a natural origin $\tau=0$, in the neighborhood of which $\xi(\tau)\to 0$ no faster than τ and $\omega/\xi\to 0$ (say as fast as τ to some power $\epsilon>0$), we find that it approaches tangency at $\tau=0$ with the world-line $u=u(0)$ of some one of the fundamental particles A_0. Hence in a world satisfying both these initial and final conditions each test particle may be considered as originating with the nebula A_0, dissociating itself from A_0 and eventually associating itself with some nebula A, as in the case $\xi=\tau$, $\Gamma=G(\zeta)/\tau$ considered by Milne and to which we return in the sequel; but it is to be noted that we are, in the general case, under no necessity of requiring that at some intermediate event the test particle acquires the velocity of light.

6. TEST PARTICLES AND GRAVITATION

We have stated in the concluding section of Part I that any gravitational theory whose basic formulation is consistent with the cosmological principle may be imposed within the frame of the general kinematical theory which is the subject of this paper. The addition of any complete gravitational theory should enable us to determine $\xi(\tau)$, which specifies the motion of the fundamental particles, and $\Gamma(\tau,\omega)$, which determines that of test particles added to the system —or it should at least enable us to express these quantities in terms of specific dynamical concepts, such as mass or energy and pressure. We here illustrate the simplest aspects of this situation by imposing two gravitational theories of widely divergent types: (*a*) the general relativistic theory of gravitation and (*b*) an extension of the Newtonian theory which is formally consistent with the cosmological principle. We postpone the discussion of Milne's kinematical-statistical theory until after the treatment of statistical systems in the concluding Part III of this investigation.

We preface these applications with an analysis of the space-time distribution of the fundamental particles which give rise to the gravitational field in which the test particle moves. For this purpose we consider the fundamental particles (nebulae) as identifiable points which are fixed in the auxiliary space (0.2), and assume without fur-

ther ado that distances and volumes in space-time are to be measured with the aid of the metric (2.1)—in units in which the instantaneous velocity of light c is unity. Now let d be the number of particles at time τ within a space-time region whose associated u-volume, as measured with the aid of the auxiliary metric (0.2), is unity; then d is actually independent of τ, for these particles are for all time represented by fixed points $\eta^a = \text{const.}$ in the u-space. But the volume, as measured by the space-time metric (2.1), occupied by these particles at time τ is $\xi^3(\tau)$, and hence their particle-density $n(\tau)$ at this time is the quotient $d/\xi^3(\tau)$—or, adopting for convenience a form in which a distinction is made between the physical dimensions of space and time,

$$n(\tau) = \frac{d}{R^3}, \tag{6.1}$$

where $R(\tau) = c\xi(\tau)$. This equation is, of course, simply the integrated form of the differential conservation law, expressed here by the vanishing of the divergence (in the sense of the metric [2.1]) of the flow-vector $n\delta_0^\mu$.

a) Einstein's theory of gravitation.—In this theory the natural kinematics, or space-time geometry, is determined with reference to the physical content of the universe through the field equations. On applying it to an idealized universe in which the cosmological postulate is satisfied, one finds that the line-element must be of the form (2.1), where $\xi(\tau)$ is determined implicitly in terms of the physical content, as described by the matter-energy tensor $T^{\mu\nu}$.[5] For the case in hand, in which account is taken only of the incoherent fundamental particles, this tensor is simply

$$T^{\mu\nu} = \rho(\tau)\delta_0^\mu\delta_0^\nu, \tag{6.2}$$

where $\rho \equiv dm/R^3$ is the mass-density obtained from (6.1) on the assumption that each fundamental particle has the same proper mass m. The resulting field equations involve, in addition, two constants—the curvature k of the auxiliary space and the cosmological constant λ—the complete specification of which can be accomplished only with the aid of further observational material; the latitude al-

[5] Cf. report referred to in n. 2.

lowed by the theory is already very considerably restricted, as shown by the recent investigations of Hubble and Tolman.[6]

Secondly, we must determine the quantity $\Gamma(\tau, \omega)$, which characterizes the absolute acceleration A^μ of a test particle in such a universe. Now, in the relativistic theory the world-line of a free test particle is a geodesic $A^\mu = 0$ of the metric (2.1); hence, if we retreat somewhat from our idealized description in terms of a continuum of fundamental particles (as implied already in the derivation of [6.1] above), we may say that the motion of a test particle between collisions is geodesic. But (4.4), (5.3) then tell us that

$$\Gamma(\tau, \omega) = 0, \quad \omega = \text{const.}, \tag{6.3}$$

and the integration of the equations of motion is reduced to the quadrature (5.5), in which $\xi(\tau)$ is known from the solution of the field equations and ω is a constant. If we insist, on the other hand, upon retaining the formulation in terms of a continuum, the function $\Gamma(\tau, \omega)$ must be obtained from a more profound analysis of the direct interaction forces and of the conservation laws implied by the field equations; while this attack might prove of interest in certain of the statistical problems of cosmology, we pass over it here as belonging more properly to the theory of relativity itself.

b) Relativistic extension of the Newtonian theory.—Whereas Einstein's theory of gravitation proceeds by determining the natural space-time geometry with reference to the physical constitution of the system under examination, the Newtonian theory considers space-time as strictly *a priori* in the Kantian sense. In order to fit such a theory into the kinematics developed in these pages, it is clear that the fundamental line-element (2.1) must be that of a completely (not merely spatially) homogeneous and isotropic manifold, in the sense that there shall be no intrinsic distinction between any two timelike directions at an event or between any two spacelike directions. As such, the problem of the kinematical background falls within the scope of the Helmholtz-Lie investigations, applied to a four-space of signature -2 instead of to a positive-definite three-

[6] E. Hubble and R. C. Tolman, "Two Methods of Investigating the Nature of the Nebular Red-Shift," *Ap. J.*, **82**, 302–337, 1935.

space. But this means that (2.1) must define a manifold of constant Riemannian curvature K, and this in turn that $\xi(\tau)$ must satisfy the equations[7]

$$\left(\frac{d\xi}{d\tau}\right)^2+k+K\xi^2=0\,,\qquad \frac{d^2\xi}{d\tau^2}+K\xi=0\,. \tag{6.4}$$

Examination of these equations reveals the fact that adherence to this program reduces the possible modes of motion of the fundamental observers to six types, defined by $k=-1$, $K<$, $=$, or >0; $k=0$, $K<$ or $=0$; $k=+1$, $K<0$.[8] We remark in passing that of these the case $k=0$, $K=0$ describes the Minkowski world (0.5) with stationary observers; the case $k=-1$, $K=0$, the form of the Minkowski world implied by Milne's kinematical theory; and the case $k=0$, $K<0$, the stationary form of the de Sitter universe. The case of most interest for the remainder of this section is the Lanczos form

$$k=+1,\qquad K=\frac{-1}{a^2}\,,\qquad \xi(\tau)=a\cosh\left(\frac{\tau}{a}\right) \tag{6.5}$$

of the de Sitter universe; it alone defines a universe in which the spaces $\tau=$const. are closed, and hence in which the total mass of a homogeneous distribution is finite.

A gravitational theory suitable for general use in any of these six universes should enable us to obtain the equations of motion of a test particle in the presence of any given distribution of matter, subject, of course, to certain conditions of finiteness the quantities involved. In the Newtonian theory this is accomplished by introducing the intermediary notion of the *gravitational field* and by express-

[7] Obtained from eqs. (A.7) and (B.3) of the report referred to in n. 2 above. Although the second of our equations (6.4) is in general a consequence of the first, it must be called upon explicitly to rule out, e.g., the Einstein universe $\xi=$const., $k=+1$.

[8] Note that from the standpoint of the general relativistic theory these are simply the six possible forms of "empty" space-time, with cosmological constant $\lambda=-3K$, as classified under E (energy)$=0$ in Table 2, p. 77, of our report referred to in n. 2 above; for their explicit determination see *ibid*., eqs. (6.1), (6.6)–(6.8). I take this opportunity to acknowledge my indebtedness to the late Professor de Sitter for calling my attention to a misstatement made in this connection: two lines above eq. (6.8) I referred to "the Minkowski world (for $\lambda=0$, $k=\pm1$)," whereas I should have referred to the cases $k=0$ or -1.

ing the equations of motion with its aid. We indicate here briefly how this same procedure might be adapted to our present needs with a minimal deviation from the Newtonian lines, and illustrate our general scheme by a more precise formulation which proves applicable to the cosmological problem, provided the total mass involved is finite.

Let the field be specified by a scalar "potential" $V(\tau, \eta^a)$ obtained from the scalar density $\rho(\tau, \eta^a)$ of the given distribution by means of a second-order linear differential equation, which reduces to the classical Poisson equation in that approximation in which the velocities and linear dimensions involved may be considered as small in comparison with the velocity c of light and the length $c|K|^{-\frac{1}{2}}$. Now this scalar equation must, except for the term involving ρ, be invariant in form under the transformations of the group G_{10} of motions admitted by any space-time (2.1) satisfying the uniformity conditions (6.4); as such it can, incidentally, contain the second derivatives of V only in the combination

$$\Delta V \equiv g^{-\frac{1}{2}} \frac{\partial}{\partial \eta^\mu}\left(g^{\frac{1}{2}} g^{\mu\nu} \frac{\partial V}{\partial \eta^\nu}\right), \tag{6.6}$$

representing the generalized Laplacian computed from the metric (2.1). In virtue of this invariance the potential must exhibit the same properties symmetry as the distribution to which it is due; hence, for the spatially homogeneous distribution (6.1) at rest in the co-ordinate system η^a the potential, if it exists, can at most be a function $V(\tau)$ of the cosmic time τ of the event at which it is computed.

Although the foregoing general outline suffices for our further purposes, we digress for a moment to illustrate it by exhibiting one possible extension of the Newtonian potential theory in a form applicable at once to all six world-types found above, and which reduces rigorously to the classical theory for any stationary distribution $\rho(\eta^a)$ in the Minkowski world $k=0$, $K=0$. The equation satisfied by the potential function $V(\tau, \eta^a)$ in this proposed extension is

$$\Delta V - 2KV = -4\pi c^2 \gamma \rho \,, \tag{6.7}$$

where γ is the Newtonian constant of gravitation and $\rho(\tau, \eta^a)$ is the mass-density of the given distribution; for a stationary field in the

stationary form of the Minkowski universe it reduces to the Poisson equation, as implied above. Its solution can be effected with the aid of the elementary potential

$$U(\tau, \eta^{\alpha}; \eta_0^{\alpha}) = -\frac{\gamma m}{R(\tau)\sigma(u)}, \tag{6.8}$$

representing the field at $E(\tau, \eta^{\alpha})$ due to a particle of proper mass m fixed at a point η_0^{α} whose u-distance from the event E is u; σ is here that function of k and u defined by (0.4). Note that the denominator of U is therefore precisely that distance function of the two events defined by Tolman and by Whittaker in their investigations of radiation phenomena in space-time.[9] On applying the potential theory implied by (6.7), (6.8) to the homogeneous distribution (6.1) of particles of equal mass m, it can be shown that the resulting potential $V(\tau)$ is finite only in the spatially closed Lanczos-de Sitter universe (6.5), in which it is found to have the form

$$V(\tau) = -\frac{bc^2}{a} \operatorname{sech}\frac{\tau}{a}, \tag{6.9}$$

where $b = 4\pi\gamma dm/c^3$ is a constant intrinsic to the problem and having the physical dimensions of time. In all other universes of the types here considered the potential theory provisionally proposed in this paragraph goes aground on the same rock of infinite mass as does the classical theory.

The second step in the extension of the Newtonian theory consists in setting up the equations of motion of a test particle in a field described by the potential $V(\eta^{\mu})$. The acceleration vector A^{μ} defined by (4.2) must here be expressed in terms of vectors and scalars intrinsic to the problem, without preference to any space-time direction as such; the space-time background is here, in contradistinction to that on which section 4 was based, completely (not merely spatially) isotropic. But this means that the only vectors and scalars which can enter into the problem are those which can be built up from the single scalar $V(\eta^{\mu})$ and the velocity $\zeta^{\mu} = d\eta^{\mu}/ds$ of the test particle—for we here assume that we are dealing with a case in which the test particle suffers no direct interaction, such as collisions, with

[9] Cf. *ibid.*, p. 87, n. 2.

the particles producing the field, and that $\rho(\eta^\mu)$ can therefore enter only indirectly through the potential V. We further require, in consonance with the Newtonian theory, that Λ^μ involve at most first derivatives of V, that it be linear and homogeneous in V and its first derivatives (thus insuring the superposition principle), and that to within quantities of relative order $(v/c)^2$ the equations of motion agree with the classical ones for a stationary distribution in the Minkowski universe $k=0$, $K=0$. It can then be shown, by an analysis so similar to that employed in section 4 above that we do not repeat it here, that these conditions imply the unique form

$$\Lambda^\mu = c^{-2}(g^{\mu\nu} - \zeta^\mu\zeta^\nu)\,\frac{\partial V}{\partial \eta^\nu} \tag{6.10}$$

for the equations of motion.[10]

Finally, we have only to show that these equations (6.10) are of the form (4.4) in the case in which the field is that due to a spatially uniform distribution of fundamental particles, and to complete their integration by solving the differential equation (5.3). This field is characterized by a potential $V(\tau)$; and upon computing the right-hand side of (6.10) with the aid of the line element (2.1), we then find that the equation (6.10) reduces to (4.4), where the acceleration func-

$$\Gamma(\tau,\,\omega) \equiv \frac{V'(\tau)}{c^2} \tag{6.11}$$

tion is a function only of τ, the prime again indicating differentiation. The solution (5.4) of the equation (5.3) is then immediately seen to be

$$\omega(\tau) = \omega_0 e^{\frac{-[V(\tau)-V(\tau_0)]}{c^2}}\,; \tag{6.12}$$

the problem of locating the particle in its trajectory is reduced to that of performing the quadrature (5.5), in which ξ and ω are now known functions of τ.

[10] Note that these equations are equivalent to the variation principle $\delta\int e^{V/c^2}ds = 0$, i.e., the trajectories are geodesics of the space with metric $dS^2 = e^{2V/c^2}g_{\mu\nu}d\eta^\mu d\eta^\nu$. To that approximation in which (6.10) reduce to the classical equations of motion, this alternative formulation reduces to Hamilton's principle. For the case $k=0$, $K=0$ in which $V=-\gamma m/r$ represents the field of a spherically symmetrical sun of mass m, these equations give rise to a retardation, of magnitude one-third the Einstein advance, in the motion of the perihelion of a planet, and to no deflection of light on passing the sun.

The considerations at the end of the previous section now enable us to conclude from (6.12) that in any case in which as $\tau \to \infty$, $\xi(\tau) \to \infty$ at least as fast and $V(\tau) \to$ any limit other than $-\infty$, each test particle is eventually to be found associated with some one of the preferred observers A. This applies in particular to the case defined by (6.5), (6.9); and, since for it the equations of motion (4.5) are unchanged upon reversing the sign of τ, we can also conclude that, as we follow back the trajectory toward $\tau = -\infty$, the test particle will be found more and more closely associated with some other preferred observer A. Similarly, for those cases in which as $\tau \to 0$, $\xi(\tau) \to 0$ no faster and $V(\tau) \to +\infty$ faster than $-c^2 \log \xi(\tau)$, the particle may be considered as having originated at $\tau = 0$ with some one A_0 of the fundamental particles.

With this we have shown that our general kinematics can be supplemented by either the general relativistic theory of gravitation or by some appropriate adaptation of the Newtonian theory to give a complete dynamical description of the motion of a test particle. In the concluding part, III, of this paper we show how the fundamental system of particles can be augmented by any statistical system satisfying the cosmological postulate, and we examine the situation arising upon again supplementing the kinematics by a gravitational theory.

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EQUILIBRIUM OF STELLAR ATMOSPHERES UNDER A TEMPERATURE GRADIENT*

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ABSTRACT

If the temperature gradient maintained by the radiation flux is smaller than the adiabatic gradient of the stellar matter and, therefore, is not sufficient to set up convection currents, the constituents of the atmosphere will be separated partially by diffusion. For all constituents participating in processes of dissociation or ionization there are remarkable deviations from the well-known exponential distribution of the partial pressures (depending only on the molecular weight), the additional terms containing the space derivatives of the thermodynamical equilibrium constants. The kinetic equivalents to these terms are stationary diffusion currents, which transport reaction energy and increase the heat conductivity, although there is no net flux of mass. This diffusion equilibrium may be realized in the atmospheres of early B stars and of stars between the types G5 and M, which ought to be free from convection.

The equilibrium of a mixture of gases in the state of dissociation or ionization under the combined influence of gravity and a temperature gradient has not yet been investigated. Milne[2] dealt with the dissociation of a gas in an external field of force along the lines of Willard Gibbs's classic memoir on "The Equilibrium of Heterogeneous Substances." The essential features of the chemical equilibrium under a temperature gradient were pointed out by Nernst,[3] who treated the dissociation under constant pressure in a reaction tube, both ends of which are maintained at different temperatures. This special case has also been treated by Dirac[4] independently. A paper by Rossby[5] regarding the equilibrium of a non-reacting mixture of gases in the temperature field of the earth's atmosphere may be quoted here too. The attempt to solve the general problem, as stated above, faces certain characteristic difficulties inherent in every treatment of systems which are not in true equilibrium but only in a stationary state. A complete thermodynamical theory of

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[2] *Proc. Cambridge Phil. Soc.*, **22**, 493, 1924.

[3] *Festschrift für Ludwig Boltzmann* (1904), p. 904.

[4] *Proc. Cambridge Phil. Soc.*, **22**, 132, 1924.

[5] *Arkiv för Matematik, Astronomi och Fysik*, **18**, Nr. 20, 1924.

stationary states is still much to be desired. An attempt will be made here to escape these difficulties by employing the conception of local thermodynamical equilibrium, but it must be admitted that the discussion remains open to the logical objection that this conception does not correspond exactly to the physical conditions in stars.

Without examination for the moment as to how far these conditions are realized in stellar atmospheres, the following assumptions are made. The atmospheres are looked on as systems in hydrostatic and in local thermodynamical equilibrium. Then the law of mass action is satisfied everywhere, and the temperature appearing later in the formulae is the material one as defined by the mean square of molecular velocity. The density is supposed to be high enough to allow an exchange by collisions between the energy of the internal degrees of freedom and the translational energy; this makes it possible to attribute a well-defined temperature to the stellar matter. The actual distribution of temperature, which is maintained by the constant net flux of radiation from the stellar interior and depends upon the variable opacity corresponding to the change in composition with the elevation, does not enter the following analysis. The first aim is to establish the relations between the temperature gradient and the gradients of partial pressure of the single constituents. For this purpose it can be assumed that an arbitrary distribution of temperature is impressed on the reacting mixture by adding an inert gas of suitable opacity. This idea does not mean any restriction of the analysis. Since the solution looked for is not to involve the actual distribution of temperature, the equations representing the gradients of partial pressure will reduce to the isothermal form, the well-known barometric-height formula, if the temperature gradient vanishes. For sake of simplification the problem is taken for a plane one with constant gravity acceleration throughout the atmosphere, and other forces such as radiation pressure or centrifugal acceleration are neglected. It is useful to distinguish between constituents (or chemical substances) and components of a system, as it is done in the theory of phase equilibria. A. Findlay[6] defines as follows: ". . . . As the components of a system there are to be

[6] *The Phase Rule* (New York, 1915).

chosen the smallest number of independently variable constituents, by means of which the composition of each phase participating in the state of equilibrium can be expressed in the form of a chemical equation."

Now the equation of hydrostatic equilibrium may be written in the form of a sum of the contributions of every particular kind of constituent,

$$\sum^{i} \frac{dp_i}{dx} + g \sum^{i} \rho_i = 0 \; . \tag{1}$$

This equation is equivalent to a set of $i+1$ equations:

$$\frac{dp_i}{dx} + g\rho_i = \psi_i \; , \tag{2}$$

$$\sum^{i} \psi_i = 0 \; . \tag{3}$$

It is evident that the functions ψ_i vanish separately in the isothermal case, and this very fact has also been proved thermodynamically by Milne.[2] In the more general case of local thermodynamical equilibrium under a temperature gradient it will turn out that for every constituent participating in a reaction its ψ_i has to be different from null in order to satisfy the law of mass action everywhere. For any inert constituent the ψ_i vanishes. By introducing logarithms the law of mass action takes the form

$$\sum^{i} a_{ni} \, ln p_i = ln K_n \; , \tag{4}$$

where a_{ni} indicates the numbers of the elementary particles participating in the nth reaction with the equilibrium constant K_n, namely,

$$a_{n1}A_1 + a_{n2}A_2 + \cdots\cdots = a'_{n1}A'_1 + a'_{n2}A'_2 + \cdots\cdots$$

These a_{ni} in equation (4) are to be taken with positive sign for the consumed particles and with negative sign for the originating ones. Putting $\psi_i = \varphi_i p_i$, one gets from (2) and (3)

$$\frac{1}{p_i} \cdot \frac{dp_i}{dx} = \varphi_i - \frac{gm_i}{kT} \; , \tag{2a}$$

$$\sum^{i} \varphi_i p_i = 0 \; . \tag{3a}$$

Now if (2*a*) is substituted into the derivative of (4), taken with regard to the x co-ordinate (elevation in the atmosphere, positive upward),

$$\sum^{i} a_{ni}\varphi_i - \frac{g}{kT}\sum^{i} a_{ni}m_i = \frac{dlnK_n}{dx} . \tag{4a}$$

By the conservation of mass during the reaction, the second term on the left side of this equation vanishes identically. Then (3*a*) represents the demanded conservation of hydrostatic equilibrium under a temperature gradient and a set of n simultaneous equations,

$$\sum^{i} a_{ni}\varphi_i = \frac{dlnK_n}{dx} , \tag{4b}$$

expresses the fact that the reaction equilibria are satisfied throughout the atmosphere.

Equations (3*a*) and (4*b*) in the special form corresponding to the equilibrium between a certain atom (index 1) and its double molecule (index 2) lead to the conditions

$$\varphi_1 p_1 + \varphi_2 p_2 = 0 , \tag{5a}$$

$$2\varphi_1 - \varphi_2 = \frac{dlnK}{dx} . \tag{5b}$$

The functions φ_1 and φ_2 can be evaluated at once for this one-component system, giving

$$\varphi_1 = + \frac{p_2}{p_1 + 2p_2} \cdot \frac{dlnK}{dx} , \tag{6a}$$

$$\varphi_2 = - \frac{p_1}{p_1 + 2p_2} \cdot \frac{dlnK}{dx} . \tag{6b}$$

Then, with α denoting the degree of dissociation, the complete differential equations ruling the distribution of partial pressures are (substitution of 6*a* and 6*b* into 2*a*)

$$\frac{dlnp_1}{dx} = -\frac{gm_1}{kT} + \frac{1-\alpha}{2} \cdot \frac{dlnK}{dT} \cdot \frac{dT}{dx} , \tag{7a}$$

$$\frac{dlnp_2}{dx} = -\frac{gm_2}{kT} - \alpha \cdot \frac{dlnK}{dT} \cdot \frac{dT}{dx} . \tag{7b}$$

Before proceeding to more complicated systems it is very useful to interpret the state of equilibrium described by equations (7*a*) and (7*b*) in terms of the kinetic theory of gases. Chapman[7] has brought the kinetic theory of diffusion into a form which is very convenient for this purpose. He studied the internal motions in binary mixtures of gases under the influence of external forces, a temperature gradient, and locally changing relative concentrations of the gases. If the thermal diffusion, which is of minor importance, is disregarded, every gas in an atmosphere undergoes a spontaneous diffusion upward, following the diminishing pressure, and a forced one downward, following gravity. The isothermal equilibrium is marked by the fact that the two processes cancel each other and there is no resulting component of motion. The anomalous pressure gradients $\varphi_i p_i$ connected with the equilibrium of a reacting mixture under a temperature gradient maintain a stationary process of diffusion and every gas participating in a reaction has a mean component of motion. This transport of mass is counterbalanced by the reaction going on and there is no net flux of mass. The combination of equations (7*a*) and (7*b*) with Chapman's equation (10.11)[7] gives for the difference of the velocities of streaming (U_1 atom, u_2 molecule) the expression

$$U_2 - U_1 = + \frac{1+\alpha}{2} \cdot \mathfrak{D}_{12} \cdot \frac{d\ln K}{dT} \cdot \frac{dT}{dx} - \mathfrak{D}'_T \cdot \frac{dT}{dx} \, . \tag{8}$$

Since the process is a stationary one, another condition has to be imposed, namely,

$$U_1 p_1 + 2 U_2 p_2 = 0 \, . \tag{9}$$

The coefficient 2 appears because each molecule is equivalent to two atoms. Then the velocities of streaming are

$$U_1 = -(1-\alpha)\left[\frac{1+\alpha}{2} \cdot \mathfrak{D}_{12} \cdot \frac{d\ln K}{dT} - \mathfrak{D}'_T\right]\frac{dT}{dx} \, , \quad \text{(atoms ascending)} \tag{10a}$$

$$U_2 = +\alpha\left[\frac{1+\alpha}{2} \cdot \mathfrak{D}_{12} \cdot \frac{d\ln K}{dT} - \mathfrak{D}'_T\right]\frac{dT}{dx} \, . \quad \text{(molecules falling)} \tag{10b}$$

[7] *Phil. Trans. R. Soc. Lond.*, A, **217**, 115, 1917.

Here $\mathfrak{D}_{12}$ denotes the ordinary diffusion coefficient and $\mathfrak{D}'_T$ is related to the coefficient of thermal diffusion by the formula

$$\mathfrak{D}_T \cdot \frac{(p_1+p_2)^2}{p_1 p_2} \cdot \frac{1}{T} = \mathfrak{D}'_T \cdot$$

$\mathfrak{D}_{12}$ is inversely proportional to the total density, and $\mathfrak{D}'_T$ is independent of the variables of state; i.e., it is a function of only the fields of force surrounding the atom and molecule.

The conditions of mechanical and thermodynamical equilibrium are not sufficient to furnish a unique solution in systems of more than one component. If i is the number of the constituents and n the number of species of molecules, then $i-n$ is the number of the components (different sorts of atoms). To evaluate the i functions φ_i, there are only $n+1$ equations like (3a) and (4b) and the problem seems to be $(i-n-1)$-fold indeterminate. But additional conditions can be derived from the equations of continuity, corresponding to (9), which must be satisfied for every particular component. Actually the condition of hydrostatic equilibrium is split up into a set of $i-n$ conditions which make the problem definite. This may be demonstrated for a system of two components, the indices 1 and 2 referring to the two sorts of atoms, the indices 11 and 22 respectively to the double molecules, and the index 12 to the mixed molecule. Equations (3a) and (4b) take the special form

$$\varphi_1 p_1 + \varphi_{11} p_{11} + \varphi_2 p_2 + \varphi_{22} p_{22} + \varphi_{12} p_{12} = 0 \,, \tag{11}$$

$$2\varphi_1 - \varphi_{11} = \frac{dlnK_{11}}{dx} \,, \tag{12a}$$

$$2\varphi_2 - \varphi_{22} = \frac{dlnK_{11}}{dx} \,, \tag{12b}$$

$$\varphi_1 + \varphi_2 - \varphi_{12} = \frac{dlnK_{12}}{dx} \,. \tag{12c}$$

With U's denoting the velocities of streaming as before, the two equations of continuity are

$$U_1 p_1 + 2U_{11} p_{11} + U_{12} p_{12} = 0 \,, \tag{13a}$$

$$U_2 p_2 + 2U_{22} p_{22} + U_{12} p_{12} = 0 \,. \tag{13b}$$

After adding them,

$$U_1p_1+U_2p_2+2U_{11}p_{11}+2U_{22}p_{22}+2U_{12}p_{12}=0\ . \tag{14}$$

A comparison of the coefficients of (11) and (14) yields the proportion

$$\varphi_1:\varphi_2:\varphi_{11}:\varphi_{22}:\varphi_{12}=U_1:U_2:2U_{11}:2U_{22}:2U_{12}\ . \tag{15}$$

By substitution from (15), the equations of continuity (13*a*) and (13*b*) are transformed into

$$\varphi_1p_1+\varphi_{11}p_{11}+\tfrac{1}{2}\varphi_{12}p_{12}=0\ , \tag{16a}$$

$$\varphi_2p_2+\varphi_{22}p_{22}+\tfrac{1}{2}\varphi_{12}p_{12}=0\ , \tag{16b}$$

whose sum is identical with (11). The elimination of the φ_i's from the five equations (12*a*, *b*, *c*,) and (16*a*, *b*) gives

$$\left.\begin{aligned}\varphi_1=\frac{1}{N}\Big[&(p_2+2p_{22})p_{12}\cdot\frac{dlnK_{12}}{dx}+(2p_2+p_{12}+4p_{22})p_{11}\cdot\frac{dlnK_{11}}{dx}\\&-p_{12}p_{22}\cdot\frac{dlnK_{22}}{dx}\Big]\,,\end{aligned}\right\} \tag{17a}$$

$$\left.\begin{aligned}\varphi_2=\frac{1}{N}\Big[&(p_1+2p_{11})p_{12}\cdot\frac{dlnK_{12}}{dx}+(2p_1+p_{12}+4p_{11})p_{22}\cdot\frac{dlnK_{22}}{dx}\\&-p_{12}p_{11}\cdot\frac{dlnK_{11}}{dx}\Big]\,,\end{aligned}\right\} \tag{17b}$$

$$\left.\begin{aligned}\varphi_{11}=\frac{1}{N}\Big[&2(p_2+2p_{22})p_{12}\cdot\frac{dlnK_{12}}{dx}-\{p_1(2p_2+p_{12}+4p_{22})\\&+p_{12}(p_2+2p_{22})\}\cdot\frac{dlnK_{11}}{dx}-2p_{12}p_{22}\cdot\frac{dlnK_{22}}{dx}\Big]\,,\end{aligned}\right\} \tag{17c}$$

$$\left.\begin{aligned}\varphi_{22}=\frac{1}{N}\Big[&2(p_1+2p_{11})p_{12}\cdot\frac{dlnK_{12}}{dx}-\{p_2(2p_1+p_{12}+4p_{11})\\&+p_{12}(p_1+2p_{11})\}\cdot\frac{dlnK_{22}}{dx}-2p_{12}p_{11}\cdot\frac{dlnK_{11}}{dx}\Big]\,,\end{aligned}\right\} \tag{17d}$$

$$\left.\begin{aligned}\varphi_{12}=\frac{1}{N}\Bigg[(p_1+2p_{11}+p_2+2p_{22})p_{12}\cdot\frac{dlnK_{12}}{dx}+2(p_2+2p_{22})p_{11}\\ \cdot\frac{dlnK_{11}}{dx}+2(p_1+2p_{11})p_{22}\cdot\frac{dlnK_{22}}{dx}\Bigg],\end{aligned}\right\}\quad(17e)$$

$$N=\tfrac{1}{2}[(2p_1+p_{12}+4p_{11})(2p_2+p_{12}+4p_{22})-p_{12}^2]$$

Evidently this process can be extended to any number of constituents, since for every further molecule (associated from three or more atoms) there is another thermodynamical condition like (4*b*), and for any added sort of atom a new equation of continuity like (9) has to be satisfied. The velocities of streaming of the individual constituents cannot be given explicitly. Chapman's theory, developed only for binary mixtures, would have first to be generalized with regard to the resistance exerted by the other constituents on the interdiffusion of every selected pair of constituents.

Without serious modifications the preceding analysis can be applied to study the equilibrium of ionized gases. Of course the electrical forces between electrons and ions have to be considered in accordance with the work of Pannekoek,[8] Rosseland,[9] and Milne.[2] The simplest case is the equilibrium between a single sort of atom and its ions. Strictly, this is a two-component system, since it would not be appropriate to suppose from the beginning that the partial pressures of the electrons and ions must everywhere be equal and, therefore, that the equilibrium could be built up simply by splitting the equivalent number of atoms. If e denotes the charge of the electron and F the electric intensity (positive upward), caused by the tendency of the electrons to predominate at high levels, the equations of the isothermal equilibrium may be written

$$\frac{dp_a}{dx}=-\frac{gm_a}{kT}\,p_a\,,\qquad\text{(atoms)}\qquad(18a)$$

$$\frac{dp_i}{dx}=-\frac{gm_i}{kT}\,p_i+\frac{eF}{kT}\,p_i\,,\qquad\text{(ions)}\qquad(18b)$$

$$\frac{dp_e}{dx}=-\frac{gm_e}{kT}\,p_e-\frac{eF}{kT}\,p_e\,.\qquad\text{(electrons)}\qquad(18c)$$

[8] *B.A.N.*, **1**, 110, 1924.

[9] *M.N.*, **84**, 720, 1924.

For evaluating the functions φ_a, φ_i, and φ_e there are the conditions of hydrostatic and thermodynamical equilibrium, namely,

$$\varphi_a p_a + \varphi_i p_i + \varphi_e p_e = 0 , \tag{19a}$$

$$\varphi_i + \varphi_e - \varphi_a = \frac{d \ln K}{dx} , \tag{19b}$$

and two equations of continuity,

$$U_a p_a + U_i p_i = 0 , \tag{20a}$$

$$U_a p_a + U_e p_e = 0 , \tag{20b}$$

which imply that there is no resulting electric current, since

$$U_i p_i = U_e p_e . \tag{20c}$$

By proceeding as outlined above, the equations of continuity are transformed into

$$\varphi_a p_a + 2\varphi_i p_i = 0 , \tag{21a}$$

$$\varphi_a p_a + 2\varphi_e p_e = 0 . \tag{21b}$$

The elimination of the φ's gives

$$\varphi_a = - \frac{2 p_i p_e}{p_a(p_i + p_e) + 2 p_i p_e} \cdot \frac{d \ln K}{dx} , \tag{22a}$$

$$\varphi_i = + \frac{p_e p_a}{p_a(p_i + p_e) + 2 p_i p_e} \cdot \frac{d \ln K}{dx} , \tag{22b}$$

$$\varphi_e = + \frac{p_i p_a}{p_a(p_i + p_e) + 2 p_i p_e} \cdot \frac{d \ln K}{dx} . \tag{22c}$$

The equations (18*a*, *b*, *c*) and (22*a*, *b*, *c*,), together with Poisson's equation,

$$\frac{dF}{dx} = \frac{4\pi e}{kT} (p_i - p_e) , \tag{23}$$

are strictly valid whatever the local space charge may be. For much-discussed reasons the assumption of a vanishing space charge ($p_i = p_e$), which implies

$$eF = \tfrac{1}{2}g(m_i - m_e) , \tag{24}$$

must be a fair approximation throughout all the atmosphere. Milne showed that in an isothermal atmosphere no separation of electrons and ions occurs except in the uppermost layers, the pressure being smaller than 10^{-24} atmospheres or so. The method Milne used in his elaborate proof of this approximation can also be applied to the model atmosphere under a temperature gradient, with the result that here, too, equation (24) is a limiting solution of the potential problem. Nevertheless, it seems not to be profitable to go farther into these details. A comprehensive treatment of the problems of space charge would have to include the effects of stellar rotation and macroscopic magnetic fields, which are far beyond the scope of this paper. With the approximation $p_i = p_e$, which means the reduction of the system from two components to one, the complete differential equations of the equilibrium are

$$\frac{dlnp_a}{dx} = -\frac{gm_a}{kT} - \frac{p_i}{p_a + p_i} \cdot \frac{dlnK}{dx} , \tag{25a}$$

$$\frac{dlnp_i}{dx} = \frac{dlnp_e}{dx} = -\frac{1}{2}\frac{gm_a}{kT} + \frac{1}{2}\frac{p_a}{p_a + p_i} \cdot \frac{dlnK}{dx} . \tag{25b}$$

On introducing the ionization degree α, these equations may be written

$$\frac{dlnp_a}{dx} = -\frac{gm_a}{kT} - \alpha \cdot \frac{dlnK}{dT} \cdot \frac{dT}{dx} , \tag{26a}$$

$$\frac{dlnp_i}{dx} = \frac{dlnp_e}{dx} = -\frac{1}{2}\frac{gm_a}{kT} + \frac{1-\alpha}{2} \cdot \frac{dlnK}{dT} \cdot \frac{dT}{dx} , \tag{26b}$$

which reveal perfect analogy with the one-component problem of dissociation as given in (7a) and (7b). The velocities of streaming obey the same formulae (10a, b) as in the dissociation problem, but

here the atoms are falling with the velocity (10*b*), and both electrons and ions are ascending with the same velocity (10*a*).

As another example, the φ-functions of the two-component system in the state of single ionization may be given here without detailed derivation. The indices 1 and 2 refer to the two sorts of atoms, 3 and 4 to the corresponding ions, and 5 to the electrons. Since this system involves only two thermodynamical equilibria as compared with three in the case of the two-component problem of dissociation, these equations have no resemblance to the equations (17*a*, *b*, *c*, *d*, *e*).

$$\left.\begin{aligned}\varphi_1 = \frac{2}{N}\Bigg[\left\{ p_2p_3p_4\left(\frac{p_1}{p_5}-1\right) - p_3p_5(p_2+2p_4) \right\} \frac{d\ln K_1}{dx} \\ + \left\{ p_2p_3p_4 + p_1p_2p_4\left(1+\frac{p_3}{p_5}\right) \right\} \frac{d\ln K_2}{dx} \Bigg],\end{aligned}\right\} \quad (27a)$$

$$\left.\begin{aligned}\varphi_2 = \frac{2}{N}\Bigg[\left\{ p_1p_3p_4\left(\frac{p_2}{p_5}-1\right) - p_4p_5(p_1+2p_3) \right\} \frac{d\ln K_2}{dx} \\ + \left\{ p_1p_3p_4 + p_1p_2p_3\left(1+\frac{p_4}{p_5}\right) \right\} \frac{d\ln K_1}{dx} \Bigg],\end{aligned}\right\} \quad (27b)$$

$$\varphi_3 = \frac{1}{N}\left[\{p_1p_5(p_2+2p_4)+p_1p_2p_4\} \frac{d\ln K_1}{dx} + p_1p_2p_4 \frac{d\ln K_2}{dx} \right], \quad (27c)$$

$$\varphi_4 = \frac{1}{N}\left[\{p_2p_5(p_1+2p_3)+p_1p_2p_3\} \frac{d\ln K_2}{dx} + p_1p_2p_3 \frac{d\ln K_1}{dx} \right], \quad (27d)$$

$$\varphi_5 = \frac{1}{N}\left[p_1p_3\left\{ p_2+2p_4+\frac{2p_2p_4}{p_5} \right\} \frac{d\ln K_1}{dx} + p_2p_4\left\{ p_1+2p_3+\frac{2p_1p_3}{p_5} \right\} \frac{d\ln K_2}{dx} \right], \quad (27e)$$

$$N = p_5(p_1+2p_3)(p_2+2p_4)+p_1p_3(p_2+2p_4)+p_2p_4(p_1+2p_3)\,.$$

Neither equations (27) nor (17) can be transformed into a more condensed form by changing from the partial pressures to ionization and dissociation degrees, respectively. Nevertheless, such a transformation makes it clear that for a very large abundance ratio of the two components these formulae reduce to the equations describing the one-component equilibrium of the more abundant component.

In considering the question as to how far the basic assumptions

of the foregoing analysis are realized in stellar atmospheres, the results of H. Siedentopf's investigations[10] on atmospheric convection may be reviewed in brief: In the stars of the main sequence later than F0 and in the giant stars there is a convective layer below the photosphere. The lower the surface temperature of the star, the deeper lies this layer. Between B8 and F0 convection is set up in the photospheric layers; at temperatures higher than 10,000° there is no convection close to the stellar surface at all. Of course these numerical values may be changed a little by different assumptions regarding the composition of the stellar atmospheres and the coefficient of general opacity. Atmospheric convection is also to be expected at the lowest surface temperature, in late M-type stars.[11] Hence only the atmospheres of early B stars and stars between G5 and M3 or so may be quiet enough to allow the states of equilibrium as described above to be attained by diffusion. There is, indeed, no conclusive proof that these atmospheres are really in diffusion equilibrium. Since the diffusion is maintained by temperature differences, the stationary velocities of streaming can only be a small fraction of the mean thermal velocity; estimates of the diffusion velocity, with regard to the densities prevalent in stellar atmospheres, give 1 m/sec as the order of magnitude. The outward transport of reaction energy caused by the diffusion increases the heat conductivity, but is negligible compared to the radiative flux. The main feature of the diffusion equilibrium is the stratification of the atmospheric constituents according to their relative masses and the secondary influence of the φ-terms.

Some time ago it occurred to the writer that the well-known hydrogen anomaly in late-type stars might be caused, perhaps partially, by the settling of the light hydrogen atoms; and Professor H. N. Russell has suggested to him that the scarcity of heavy elements in the early B spectra might be an effect of gravitational separation of the elements. The writer has had the privilege of seeing before publication a forthcoming paper by Swings and Struve which describes departures from the predicted multiplet intensities of several *O* II lines situated within the wings of $H\gamma$. Eddington's theory of absorption lines is not able to give full account of the facts. The re-

[10] *A.N.*, **247**, 297, 1933.

[11] R. Wildt, *Zs. f. Ap.*, **9**, 176, 1934.

maining discrepancies could be explained, in the opinion of these authors, by the assumption of a stratified atmosphere, the heavier oxygen ions prevailing in the lower strata. Similar effects were found by Swings and Struve in A stars, although here the *Ca* II absorption seems to be situated at levels higher than those in which the formation of hydrogen wings takes place. Possibly this phenomenon has a quite different origin and is a true analogue to the sun's calcium chromosphere. Recently Christie and Wilson[12] gave very interesting details about the distribution of the elements in the atmosphere of the K component of ζ Aurigae. Unfortunately, the observations do not cover the whole duration of the eclipse, nor can they be reduced without an additional hypothesis. Certainly most valuable information will be derived from this eclipsing variable in future time. Thus far it seems that at least the outer parts of the atmosphere of ζ Aurigae are partially supported by radiative pressure; but there are indications of a different type of equilibrium in the inner atmosphere.

All current theories of stellar line intensities are based on the assumption that the atmospheres are well mixed from top to bottom, which is quite appropriate in the light of the insight recently gained into the rôle that convection plays at the surface of all stars between B8 and G5 or so. As for the earlier and later types, it would be interesting to compare the observed intensities with the theoretical ones calculated by integration of the equations of diffusion equilibrium. Such a numerical integration would be a very troublesome enterprise, because the temperature gradient is a function of the local chemical composition and has to be evaluated anew for every step of the integration. Unsöld's[13] numerical integrations of well-mixed atmospheres could be accomplished by successive approximations of the temperature distribution. His results ought to be reliable as to the solar atmosphere, which evidently is stirred up thoroughly by convection. The numbers of hydrogen atoms above the photosphere of a K giant evaluated by Unsöld are in qualitative accordance with the "departures from thermodynamical equilibrium" as observed by Adams and Russell, Payne, and others. A

[12] *Mt. W. Contr.*, No. 519; *Ap. J.*, **81**, 426, 1935.

[13] *Zs. f. Ap.*, **8**, 225, 1934.

calculation of the K spectrum on the basis of diffusion equilibrium would doubtless reveal still larger departures and hence may give an even better representation of the observations.

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THE SYSTEM OF THE HDC DURCHMUSTERUNG MAGNITUDES

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ABSTRACT

The photovisual magnitudes of 2730 stars between the eighth and eleventh magnitudes and north of $-30°$, derived at the McCormick Observatory, were used as a standard system with which to compare the BD, SD, and CoD magnitudes reduced to the HDC system by means of the tables in *Harvard Annals*, **72**, Nos. 6 and 7 (referred to as HDCDM system). Next, the HDCDM magnitudes were compared anew with the photovisual magnitudes derived by Miss Payne in Harvard standard regions A, B, C, and D and with the photovisual magnitudes derived by Ross and Zug for the Eros comparison stars. Finally, they were compared with the photovisual magnitudes determined at the Mount Wilson Observatory in the zone north of $+80°$. All four comparisons, in agreement with the conclusion of Seares, Sitterly, and Joyner, show that the scale correction to reduce the HDCDM magnitudes to the IPv system is negligibly small. When the various photovisual magnitudes are reduced to the zero point and to the color of the IPv system, the HDCDM magnitudes are well represented by the formula:

$$\text{IPv} = \text{HDCDM} + 0\overset{\text{M}}{.}15 - 0.2 \text{ C.I.}$$

Since the mean color index for the stars between the eighth and eleventh magnitudes is about $0\overset{\text{M}}{.}6$, this indicates that, on the average, the correction on the right-hand side of the equation is practically zero.

I. INTRODUCTION

Photovisual magnitudes of 18,000 faint stars fairly uniformly distributed over the sky north of $-30°$ were recently determined at this observatory in connection with a proper motion investigation.[1] Although the accidental error of the individual magnitudes is fairly large (P.E. $= \pm\overset{\text{M}}{.}12$), considerable care was taken to insure that the system of the magnitudes should be the IPv system. However, an independent supplementary investigation indicated that the small corrections given in Table I should be applied to the McCormick magnitudes printed in the catalogue, in order to reduce them to IPv magnitudes.

In order to determine the color coefficient, 180 exposures of the North Polar Sequence were used. The value -0.03 ± 0.04 was obtained from the deviations of the observed magnitudes of the red stars 8r, 11r, 12r, and 21 from the average run of the reduction curve

[1] Van de Kamp and Vyssotsky, *Proc. Nat. Acad.*, **21**, 419, 1935; also to be published *in extenso* in Vol. **7** of the *Publications of the Leander McCormick Observatory*.

of the white stars. Accordingly, no color corrections were applied to the observed magnitudes.

Of these McCormick magnitudes, 2730 were compared with Durchmusterung magnitudes reduced to the Harvard photometric system by means of the tables in *Harvard Annals*, **72**, Nos. 6 and 7, which we shall refer to hereafter as the HDCDM system. It was immediately evident that except for color coefficient there is very little difference between the McCormick and the HDCDM systems. This was somewhat unexpected, since Miss Payne[2] had concluded that there is a 10 per cent scale error in the HDCDM system, and Ross and Zug[3] had found a scale error of 34 per cent; on the other hand, Seares, Sitterly, and Joyner had found practically no scale error.[4]

TABLE I

McC	IPv − McC
$8^{\mathrm{M}}8$	$+0^{\mathrm{M}}06$
10.1	+ .01
11.0	− .03
11.9	−0.06

The cause of these apparently contradictory results was found to be in the method of discussion. Usually, when comparing different systems of magnitudes with small accidental errors, the method is a relatively unimportant matter; but it becomes very important when the accidental errors are large, as in the case of the Durchmusterung magnitudes, where the probable error of one magnitude is not far from $\pm 0^{\mathrm{M}}3$. Thus, grouping the Ross and Zug material as they did, according to Yerkes magnitudes, we find, as they did, a difference of 34 per cent between the Yerkes scale and the HDCDM scale; but, grouping the material according to the HDCDM magnitudes, we find practically no difference in the scales. The second method is the correct one to be used when it is desired to find the most probable corrections to the HDCDM magnitudes in order to reduce them to the Yerkes system.[5] Such a great difference in slope

[2] *Harvard Ann.*, **89**, 73, 1932. [3] *A.N.*, **239**, 289, 1930. [4] *Ap. J.*, **72**, 341, 1930.

[5] See, for instance, C. V. L. Charlier, *Die Grundzüge des Mathematischen Statistik*, p. 91, 1920.

between the two regression curves of a correlation diagram always occurs when the scattering of the points is large. Neither regression curve is to be considered as the so-called "true relation" between the two systems of magnitudes. But in the practical question of finding the most probable IPv magnitude corresponding to a given HDCDM magnitude, we are not concerned with that problem. Furthermore, no corrections have been made for "multitude and completeness."[6]

II. COMPARISON OF THE HDCDM MAGNITUDES WITH THE McCORMICK PHOTOVISUAL MAGNITUDES

Color coefficient.—As Miss Payne has shown,[2] the HDCDM magnitudes are affected by a considerable color equation. In order to find the relative color coefficient of the HDCDM magnitudes with

TABLE II

HDCDM	B0 to F2		F5 to K2		K5 to M	
	Δ	No. of Stars	Δ	No. of Stars	Δ	No. of Stars
$8^{M}.7$	$+0^{M}.061$	72	$-0^{M}.044$	81	$-0^{M}.400$	14
9.2	+ .006	141	.150	196	.213	39
9.7	+ .030	173	.054	307	.141	37
10.2	+ .011	185	.060	384	.333	52
10.6	−0.165	124	−0.206	310	−0.479	62

respect to the McCormick magnitudes, the magnitude difference, Δ = McC − HDCDM, was formed for 2177 stars of known spectra. The mean values of Δ for various subdivisions of the material are shown in Table II. As is readily seen, there is a well-pronounced color effect. In order to derive the color coefficient, we must know the average color index for each spectral group. Giving consideration to the color indices derived from the same material in low latitudes by Miss Williams,[7] we have adopted as the mean color index for all latitudes the following values:

Group	B0 to F2	F5 to K2	K5 to M
Adopted mean C.I.	$+0^{M}.25$	$+0^{M}.70$	$+1^{M}.50$

[6] See *Handbuch der Astrophysik*, **5**, 336, 1932.

[7] *Ap. J.*, **79**, 395, 1934.

Combining these values with the data of Table II, we find that the relative color coefficient is −0.22 and that the color corrections to be applied to the mean differences of Table II are:

Group	B0 to F2	F5 to K2	K5 to M
Adopted mean color correction	$+0^{M}_{.}055$	$+0^{M}_{.}155$	$+0^{M}_{.}332$

Scale and zero point.—The weighted mean of the differences, corrected for color, are given in Table III. The faintest group ($10^{M}_{.}6$)

TABLE III

HDCDM	$(McC-HDCDM)_{corr}$	No. of Stars
$8^{M}_{.}7$	$+0^{M}_{.}10$	167
9.2	+ .04	376
9.7	+ .10	517
10.2	+ .08	621
(10.6	−0.08	496)

in Tables II and III is seriously affected by selection, for on the average only the brighter stars of the HDCDM group $10^{M}_{.}6$ were bright enough to have had their spectra classified. Thus, on the average, the McCormick magnitude will be brighter for the stars as selected

TABLE IV

HDCDM	$(McC-HDCDM)_{corr}$	No. of Stars
$9^{M}_{.}3$	$+0^{M}_{.}20$	35
9.7	+ .18	89
10.2	+ .18	137
10.7	+ .08	205
11.2	+ .10	54
(11.8	−0.08	33)

than it would have been with a random selection. This will tend to make the value of Δ for this group less than its true value, as an examination of the tables will show in fact is the case. For this reason the faintest group of Table III has been omitted from further discussion. The same effect is well pronounced in the material of Miss Payne and of Ross and Zug (see Sections III and IV).

For 520 stars no spectra were known. Assuming an average color index of $0^{M}.6$ for these stars and a color coefficient of 0.22, we have the differences, corrected by $+0^{M}.132$ for color, shown in Table IV. The last line again shows the effect of selection, since the McCormick magnitudes for many regions investigated reach their limit around $12^{M}.0$; consequently, it was rejected.

TABLE V

Pv−HDCDM

HDCDM	McCormick		Harvard		Yerkes		Mt. Wilson	
	Diff.	No.	Diff.	No.	Diff.	No.	Diff.	No.
$6^{M}.7$	....	....	$+0^{M}.06$	52	....	....	$+0^{M}.09$	33
7.2	....	....	+ .05	80	($+0^{M}.03$	15)	+ .15	63
7.7	....	....	− .04	71	.24	55	− .12	58
8.2	....	....	− .08	81	.22	94	− .06	18
8.7	$+0^{M}.10$	167	+ .07	99	.32	169	+ .05	27
9.2	.05	411	+ .10	84	.35	173	+ .23	34
9.7	.11	606	+ .12	62	.29	95	+ .20	41
10.2	.10	758	(−0.17	53)	(+0.00	19)	+0.11	13
10.7	.08	205	....	....	....	....	....	......
11.2	+0.08	54	....	....	....	....	....	......

Combining Tables III and IV, weighted according to the corresponding numbers of stars, we obtain the figures given in column 2 of Table V. These were analyzed by a least-squares solution for scale and zero-point corrections, with the following results:

$$\text{Scale difference} = +0^{M}.010 \pm 0^{M}.014 .$$

$$\text{Zero-point correction (for } 10^{M}.0) = +0^{M}.096 \pm 0^{M}.008 .$$

Thus, the scale difference is negligible and the zero-point correction is small. In other words,

$$\left.\begin{array}{l}\text{McC}-\text{HDCDM} = +0^{M}.10 + 0.01(\text{HDCDM} - 10^{M}.0) - 0.22\ \text{C.I.} \\ \qquad\qquad\qquad\quad \pm 0.01 \pm 0.01\end{array}\right\} \quad (1)$$

For a star of average color the correction is close to zero.

III. COMPARISON OF THE HDCDM MAGNITUDES WITH THE PHOTOVISUAL MAGNITUDES IN HARVARD STANDARD REGIONS

Miss Payne has published the photovisual magnitudes of stars in the Harvard standard regions[8] and compared them with the HDCDM system.[2] For the sake of uniformity with the procedure of the preceding section, it is important to use the same regression curve in the comparison, namely, the regression of HPv magnitudes on HDCDM magnitudes. The HDCDM magnitudes were corrected by the color corrections given by Miss Payne[2] as follows:

B0 to F0	0.0	K5	−0.3
F2 to G5	− .1	Ma	.4
K0 to K2	−0.2	Mb	−0.5

The differences ($HPv - HDCDM_{corr}$), grouped with respect to HDCDM magnitudes, are given in column 4 of Table V. Rejecting the faintest-magnitude group on account of selection, as explained in Section II, we derive the following relation:

$$\left.\begin{aligned} HPv - HDCDM = +0^{M}_{.}03 &+ 0.03(HDCDM - 8^{M}_{.}0) + \text{color correction} \\ \pm 0.03 &\pm 0.02 \end{aligned}\right\} \quad (2)$$

Both scale and zero-point corrections are negligible.

IV. COMPARISON OF THE HDCDM MAGNITUDES WITH THE YERKES PHOTOVISUAL MAGNITUDES OF EROS COMPARISON STARS

Ross and Zug have published photovisual magnitudes for 636 comparison stars for Eros and have compared them with the HDCDM magnitudes.[3] They grouped the magnitude differences according to the Yerkes magnitudes, that is, they used the regression of HDCDM on the Yerkes magnitudes, which gives us the most probable value of the HDCDM magnitude for a given Yerkes magnitude; whereas we want to know the most probable value of the Yerkes magnitude corresponding to a given HDCDM magnitude. Each HDCDM magnitude was corrected for color as follows:[9]

B0 to F2	0.0	K0 to K5	−0.2
F5 to G5	−0.1	K7 to M	−0.3

[8] *Harvard Ann.*, **89**, 1, 1931.

[9] *Ap. J.*, **73**, 37, 1931.

Column 6 in Table V gives the mean difference ($Y - HDCDM_{corr}$) grouped according to HDCDM magnitude. The figures in the first and last lines of this column show the effect of selection and were consequently rejected. A least-squares solution of the remaining five equations gives:

$$\left.\begin{array}{l} Y - HDCDM = +0\overset{M}{.}28 + 0.05(HDCDM - 8\overset{M}{.}5) + \text{color correction} \\ \qquad\qquad\quad \pm 0.02 \pm 0.02 \end{array}\right\} \quad (3)$$

The scale difference is again very small.

V. COMPARISON OF THE HDCDM MAGNITUDES WITH THE MOUNT WILSON PHOTOVISUAL MAGNITUDES OF THE NORTH POLAR ZONE

At the suggestion of F. H. Seares a comparison of the HDCDM magnitudes was made with the photovisual magnitudes of 287 stars found in the Mount Wilson catalogue of magnitudes and colors in the zone $+80°$ to $+90°$.[10] The following color corrections derived from the same material were applied to the HDCDM magnitudes:

Mt. Wilson C.I.	Color Correction	Mt. Wilson C.I.	Color Correction
Up to +0.2	0.0	+1.0 to +1.4	−0.3
+0.2 to +0.6	− .1	+1.4 and over	−0.4
+0.6 to +1.0	−0.2		

The differences ($IPv - HDCDM_{corr}$) are given in column 8 of Table V.

A least-squares solution of the eight equations gives

$$\left.\begin{array}{l} IPv - HDCDM = +0.07 + 0.04(HDCDM - 8\overset{M}{.}0) + \text{color correction} \\ \qquad\qquad\qquad \pm 0.03 \pm 0.03 \end{array}\right\} \quad (4)$$

The scale difference is of the order of its probable error.

VI. REDUCTION OF THE HDCDM MAGNITUDES TO THE IPV SYSTEM

Now, knowing the relation of McC, HPv and Y to IPv, we may obtain several independent reductions of HDCDM to IPv. We have from Table I:

$$IPv - McC = +0\overset{M}{.}05 - 0.04(McC - 9\overset{M}{.}0) . \quad (5)$$

[10] F. H. Seares, F. E. Ross, and M. C. Joyner, *Magnitudes and Colors of Stars North of +80°*, 1935.

We find from Miss Payne's work[11] that

$$\text{IPv}-\text{HPv}=+0^{\text{M}}_{.}04+0.10\text{ C.I.} \tag{6}$$

And, according to Seares, Sitterly, and Joyner,[12]

$$\text{IPv}-\text{Y}=-0.06\text{ C.I.} \tag{7}$$

Re-writing (1), (2), (3), and (4) so that the scale correction is zero at $9^{\text{M}}_{.}0$, instead of at $10^{\text{M}}_{.}0$, $8^{\text{M}}_{.}0$, $8^{\text{M}}_{.}5$, and $8^{\text{M}}_{.}0$, respectively, and combining (1) with (5), (2) with (6), and (3) with (7), we have:

$$\text{IPv}-\text{HDCDM}=+0.14-0.03(\text{HDCDM}-9^{\text{M}}_{.}0)-0^{\text{M}}_{.}22\text{ C.I.}$$
from 2201 McCormick stars

$$\text{IPv}-\text{HDCDM}=+0.10+0.03(\text{HDCDM}-9^{\text{M}}_{.}0)-0.15\text{ C.I.}$$
from 529 stars in Harvard standard regions

$$\text{IPv}-\text{HDCDM}=+0.30+0.05(\text{HDCDM}-9^{\text{M}}_{.}0)-0.25\text{ C.I.}$$
from 586 Eros comparison stars (by Ross and Zug)

$$\text{IPv}-\text{HDCDM}=+0.12+0.04(\text{HDCDM}-9^{\text{M}}_{.}0)-0.25\text{ C.I.}$$
from 287 stars north of $+80^{\circ}$

We may add to this the following relation, obtained by Seares, Sitterly, and Joyner from 170 Eros comparison stars (excluding CPD stars):

$$\text{IPv}-\text{HDCDM}=+0^{\text{M}}_{.}04+0.0(\text{HDCDM}-9^{\text{M}}_{.}0)$$ (taking no account of color coefficient) .

The five determinations are satisfactorily accordant. The high values of the constant term as derived from the Yerkes material may possibly be traced to a right acension error in the *Harvard Annals*, **72,** tables, since all of the Ross and Zug stars lie in a narrow band of right ascension between $5^{\text{h}}_{.}5$ and $10^{\text{h}}_{.}5$. On the other hand, if Seares, Sitterly, and Joyner had been able to take account of color coefficient, their constant term would have been increased by not less than one-tenth of a magnitude. With these considerations in mind,

[11] *Harvard Ann.*, **89,** 105, 1935.

[12] *Ap. J.*, **72,** 332, 1930.

the following expression may be taken as a close representation of the reduction of the HDCDM to the IPv system:

$$\mathrm{IPv}-\mathrm{HDCDM}=+0^{\mathrm{M}}.15-0.2\ \mathrm{C.I.}$$

Thus, since the mean color of stars between the eighth and eleventh magnitudes is about $0^{\mathrm{M}}.6$, the average correction to reduce a set of HDCDM magnitudes to the IPv system is practically zero.

I am indebted to Dr. Emma T. R. Williams for discussions of various statistical problems as they appeared in the present investigation. My thanks are also due to Professor F. H. Seares and Dr. Cecilia Payne Gaposchkin, who very kindly read the manuscript and made helpful suggestions and criticisms.

Leander McCormick Observatory
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ULTRA-VIOLET STELLAR SPECTRA WITH ALUMINUM-COATED REFLECTORS

IV. THE SPECTRUM OF α BOÖTIS

R. WILLIAM SHAW

ABSTRACT

The ultra-violet spectrum of α Boötis has been investigated between λ 3950 and λ 3150. Nearly all of the 336 lines measured have been identified. Important lines of *aluminum*, *chromium*, and *copper* were observed. *Iron* and *nickel* have a number of complete multiplets the characteristics of which are admirable for intensity work. Much of the general decrease in intensity of the ultra-violet region may be due to the strong resonance lines of *nickel*, *titanium*, and *vanadium*. *Sodium* is characterized by the second principal series doublet at λ 3302. It is suggested that this pair of lines may be useful in studies of interstellar matter.

The *NH molecular* spectrum at λ 3360 was identified and shown to be free from serious blending except for the Ti^{+} line at λ 3361. Suggestions as to the importance of this band in astrophysical work are made.

The use of the words *near*, *far*, and *extreme* as related to ultra-violet stellar work is discussed in a note. It is suggested that *near* refer to the region from H and K to λ 3650, that *far* refer to the region from λ 3650 to λ 2950, and that *extreme* refer to all regions beyond λ 2950.

The far[1] ultra-violet spectra of the late-type stars are of fundamental importance in astrophysical problems because of the many atomic resonance and low energy-level lines and diatomic spectra, such as *NH* and *OH*, which may be expected to be observed in the region λλ 3000–3650.

The spectrograms for an investigation of the spectrum of α Boötis, Ko, were obtained during the summers of 1933 and 1934 at the

[1] The words "near," "far," and "extreme" seem destined to play an important part in statements concerning the ultra-violet spectra of stars. It is not necessarily desirable to repeat precise wave-length regions in a report if sufficient clarity can be maintained by the use of a word. Already some confusion is arising from the use of the word "far" in the older papers, where it refers not to the new region made available by aluminized mirrors but to a region of wave-lengths somewhat shorter than the violet. The suggestion is therefore made that "near" shall refer to the portion of the ultra-violet from the calcium H and K lines to λ 3650, the limit of the Balmer series; that "far" shall refer to the region from λ 3650 to λ 2950, the beginning of the ozone absorption and the present natural limit of stellar spectra; and that "extreme" shall refer to such portions of the spectrum beyond λ 2950 as may be observed under special conditions, as in the solar work at λ 2100 by Meyer, Schein, and Stoll (*Nature*, **134**, 535, 1934). Careful consideration of these divisions seems to indicate that they conform most readily to the greatest number of stellar characteristics, as well as to the wave-length ranges covered by the older work in the ultra-violet.

Lowell Observatory,[2] with a two-prism quartz spectrograph[3] and a 15-inch aluminized mirror. The first set of spectrograms formed the basis of a preliminary survey. With the results of this survey in mind a second set of plates was secured in which more excellent definition was attained in certain parts of the spectrum, and the ultra-violet limit was extended 150 A. Also, a series of standard spectra of certain elements whose presence in α Boötis was substantiated by the preliminary survey were photographed with the same instrument used for the stellar spectra. The standard spectra thus obtained included *Al*, *C*, *Cd*, *Co*, *Cr*, *Fe*, *Mg*, *Mn*, *Ni*, *Ti*, and *V*. These spectra were especially helpful in the more difficult cases of identification. Due consideration was given, of course, to the difference in excitation between the terrestrial and the stellar spectra. Particular attention was paid to multiplet and intensity relations in all identification work. In this connection the *Multiplet Table of Astrophysical Interest* by C. Moore was of major helpfulness. After assignments had been made and the series and multiplet relations checked, a comparison with identifications for similar solar lines in the *Revised Rowland* was carried out. The final wave-lengths, estimated intensities, and identifications are given in Table I for the region λλ 3150–3950. The spectrum was actually observed to about λ 3100. However, since it was very weak in the far ultra-violet, trustworthy measures could not be obtained beyond λ 3150.

Data are included to the longer wave-lengths as far as the H and K lines, since information on the near ultra-violet spectrum of Arcturus seems to be rather scattered and a reasonable range of wave-lengths must be included for the accurate establishment of multiplet relations for the various absorbing atoms.

ATOMIC SPECTRA

Aluminum is characterized in α Boötis by the two strong resonance lines arising from the transition 3^2P-4^2S. The *calcium* spectrum appears to be quite poor in lines, except for the H and K lines of Ca^+ and certain lines of the multiplets 4^2P-5^2S and 4^2P-4^2D which may be present but at best are badly blended. Lines of *cobalt* involve five multiplets. The intensities of the lines are quite low, and some

[2] S. L. Boothroyd, *Ap. J.*, **80**, 1, 1934. [3] R. W. Shaw, *ibid.*, **82**, 87, 1935.

TABLE I

LINES IN α BOÖTIS BETWEEN λ 3150 AND λ 3950

λ	Intensity	Identification
3146.90	4	–, 7.06 *Co*
48.08	0	**.04 *Ti*⁺**, .19 *Mn*
50.15	5	.30 *Fe*
52.22	0	.25 Ti^+
53.37	0	**.21 *Fe***, .75 *Fe*
54.51	3	.20 Ti^+, .67 *Co*, .78 *Co*
57.07	0	.03 *Fe*
59.33	v.b.	(Ca^+, *Co*, *Ni*)
62.71	0	.57 Ti^+, .80 Fe^+
64.84	2	.83 V^+, .90 Ti^+
66.50	2	.44 *Fe*
68.90	6b	.52 Ti^+, .85 *Fe*
73.35	1	.41 *Fe*
75.01	2	.04 *Fe*
76.37	0	.30 *Ni*, .35 *Fe*
77.79	2	–, 8.01 *Fe*
79.23	2	–, .33 Ca^+
80.44	1	.22 *Fe*, .74 *Fe*
81.75	3	(Ca^+, *Fe*, *Ni*)
83.32	1	.26 *Ni*, .41 *V*, .99 *V*
84.43	0	.38 *Ni*
85.28	0	.33 Fe^+, .40 *V*
86.29	1	.46 *Ti*
88.47	3	.54 *Fe*, .82 *Fe*
92.11	7b	**.00 *Ti***, .81 *Fe*
94.57	0	–, .58 Ti^+
97.37	0	.12 *Ni*, .54 Ti^+
99.97	5b	.52 *Fe*, **.92 *Ti***, .47 *Fe*
3202.72	3	.38 *V*, .54 Ti^+, .67 *Fe*
04.00	1	3.83 *Ti*, –
05.79	3	.40 *Fe*, –
07.24	3	.08 *Fe*, .41 *V*
08.89	4	....
10.67	3	–, .83 *Fe*
12.21	2	.00 *Fe*, –
14.91	3	.77 Ti^+
17.28	5	.07 Ti^+, .39 *Fe*
18.37	0	.27 Ti^+
19.71	3	**.60 *Fe***, .80 *Fe*
22.01	1	.08 *Fe*
22.98	3	.85 Ti^+
24.68	0	.63 *Co*, **.76 *Mn***
25.90	5b	.76 Ti^+, .80 *Fe*
29.19	5b	.14 *Fe*, .20 Ti^+
31.09	0	.00 *Fe*
32.85	0	.94 *Ni*
34.29	4b	.52 Ti^+, .62 *Fe*, .65 *Ni*
36.37	4	.22 *Fe*, .58 Ti^+
39.07	5	.05 Ti^+, –
41.65	1	....
43.00	1	.07 *Ni*
3244.09	0	.20 *Fe*

TABLE I—*Continued*

λ	Intensity	Identification
3247.35	4	.19 *Co*, .**55 Cu**
48.36	0	.22 *Fe*, .47 *Ni*, .61 Ti^+
49.14	0	.03 *Fe*, .19 *Fe*
50.30	0	(*Fe*, *Ni*, V^+)
51.40	0	.25 *Fe*, .85 Ti^+
52.65	1	.44 *Fe*, .89 *Fe*
54.12	3	.19 *Co*, –
55.74	2	–, .90 Fe^+
57.47	4	.23 *Fe*, .60 *Fe*
60.06	3	9.99 *Fe*, .26 *Fe*
61.59	3	.34 *Fe*, .58 Ti^+
63.57	7	.37 *Fe*, .69 Ti^+
64.84	7	.05 *Fe*, .52 *Fe*, –
67.42	1	.70 V^+
69.53	3	–, .50 *Ge*?
71.42	5	.01 *Fe*, .11 V^+, .66 Ti^+
73.93	4	.96 *Cu*
76.49	1b	.13 V^+, .47 *Co*, .**48 Fe**
78.95	3	.74 *Fe*, .93 Ti^+
82.80	0	.70 *Ni*, .83 *Ni*
85.11	0	–, .29 *Ni*
86.94	4	.77 *Fe*, .95 *Ni*
88.48	3	(*Fe*, $\mathbf{Ti^+}$)
90.64	2	.71 *Fe*, .99 *Fe*
94.97	2	....
98.20	2	.14 *V*, .14 *Fe*
99.37	1	.44 *Ti*
3302.27	6	.38 *Na*, .98 *Na*
04.54	1b	....
06.25	7	5.98 *Fe*, 6.36 *Fe*
08.25	1b	.82 Ti^+
10.42	1b	.21 *Ni*, .35 *Fe*, .50 *Fe*
12.41	1b	.32 *Ni*, .70 *Ti*
14.20	1b	(*Fe*, *Mn*, *Co*, *Ti*)
15.30	3	–, .68 *Ni*
16.63	1	....
17.94	1	–, 8.02 Ti^+
20.43	3	.26 *Ni*, –
22.84	7	.32 *Ni*, .94 Ti^+
26.61	3	–, .78 Ti^+
29.68	8	.47 Ti^+, .91 *Mg*
31.97	2	.70 *Fe*, 2.19 *Mg*
33.97	1	–, 4.15 *Co*
35.35	2	.19 Ti^+
36.60	3	.69 *Mg*
38.19	0	.20 *Fe*
40.14	3	–, .33 Ti^+
42.12	5	1.93 *Fe*, .22 *Fe*, .31 *Fe*
46.72	1	.75 Ti^+
49.35	8	.41 Ti^+
51.61	2	.53 *Fe*, .75 *Fe*
54.35	5	.**39 Co**, .39 *Fe*
56.59	1	.41 *Fe*, .68 *Fe*
3359.46	10	*NH* band

TABLE I—*Continued*

λ	Intensity	Identification
3361.30	10	**.21 Ti⁺**, .57 *Ni*, .85 *Ti*
65.87	0	**.77 Ni**, 6.17 *Ni*
69.42	4	.57 *Ni*
72.64	5	–, .80 *Ti⁺*
74.36	2	**.22 Ni**, .64 *Ni*
77.40	1	.48 *Ti*, .58 *Ti*
79.04	2	.02 *Fe*
80.75	7	.58 *Ni*, .83 *Ni*
82.46	0	.41 *Fe*
83.71	4	.69 *Fe*, .76 *Ti⁺*, .99 *Fe*
85.77	3	.95 *Ti*
88.15	7	–, .17 *Co*
91.08	4	.05 *Ni*
93.08	6	2.99 *Ni*, –
99.25	1	.24 *Fe*, .33 *Fe*
3402.24	2	.26 *Fe*
05.21	3	.08 *Co*, .17 *Co*
07.46	3	.46 *Fe*, .56 *Fe*
09.66	4	.17 *Co*, .57 *Ni*
14.39	7	3.94 *Ni*, **.77 Ni**
16.96	2	....
17.97	0	....
20.61	3	–, .75 *Ni*
23.39	4b	–, .71 *Ni*
26.79	3	....
27.52	1b	....
28.40	3	....
31.09	0	....
33.30	7	.04 *Co*, **.57 Ni**, .60 *Cr*
35.59	0	....
37.37	2	.28 *Ni*
40.77	6	**.61 Fe**, .99 *Fe*
43.80	4	.65 *Co*, **.88 Fe**
46.55	2	.27 *Ni*
49.36	2	.17 *Co*, **.44 Co**
52.91	6	.89 *Ni*
58.43	3	.46 *Ni*
61.70	5b	.66 *Ni*
65.85	2	.79 *Co*
66.10	6	5.88 *Fe*, –
68.80	1	.84 *Fe*
69.43	1	.48 *Ni*
72.40	1	.55 *Ni*
75.41	4	.45 *Fe*
76.66	4	.70 *Fe*
79.86	1	....
80.81	1	....
83.47	3	.42 *Co*, .78 *Ni*
85.35	2	.34 *Co*, .34 *Fe*
90.56	4	.57 *Fe*
93.11	4	**2.96 Ni**, –
96.81	5b	(*Co, Fe*)
97.86	3	.84 *Fe*
3498.74	0	....

TABLE I—*Continued*

λ	Intensity	Identification
3500.65	1	.56 *Fe*, **.85 *Ni***
02.27	2	.25 *Co*, .33 *Co*
05.55	0	....
06.59	2	.33 *Co*, –
10.11	4	9.84 *Co*, .33 *Ni*
13.82	1	.82 *Fe*
14.84	4	–, **5.05 *Ni***
19.78	0	.77 *Ni*
21.27	3	.26 *Fe*
23.63	3	**.43 Co,** .70 *Co*
24.24	5	.24 *Fe*, **.54 *Ni***
26.17	5	**.16 *Fe*,** .84 *Co*
27.78	0	.79 *Fe*
29.73	0	.04 *Co*, .74 V^+, .82 *Co*
32.88	2	....
33.01	1	.01 *Fe*
37.80	2	.73 *Fe*, .89 *Fe*
38.32	0	.31 *Fe*
40.90	0	.71 *Fe*, .80 *Fe*
42.97	0	....
45.36	0	.20 V^+, .64 *Fe*
47.98	2	.79 *Mn*, 8.02 *Mn*, 8.18 *Ni*
50.39	0	....
54.16	2b	3.74 *Fe*, **.12 *Fe***
55.00	3	4.94 *Fe*
56.58	1	....
58.53	3	.52 *Fe*
61.69	2	.75 *Ni*
65.40	7	–, .38 *Fe*
69.76	7	–, .80 *Mn*
70.18	4	.13 *Fe*
71.73	0	–, **.87 *Ni***
73.72	0	(*Cr*, *Fe*)
75.40	0	.36 *Co*, .37 *Fe*
78.42	5	.39 *Fe*, .68 *Cr*
81.18	7	.19 *Fe*
85.27	3	.16 *Co*, .34 *Fe*
86.69	1	.54 *Mn*, –
87.09	2	6.99 *Fe*, 7.33 *Fe*
89.43	2	.45 *Fe*
93.37	4	–, **.49 *Cr***
95.53	0	....
97.78	0	.70 *Ni*
99.13	0	.13 *Fe*
3601.98	2	....
02.23	1	.28 *Ni*
05.37	2	.33 *Cr*, .36 *Co*
06.53	0	.53 *Fe*
08.86	6	.86 *Fe*
10.36	1	.27 *Mn*, .46 *Ni*
13.19	2	2.74 *Ni*, –
15.85	0	....
18.78	7	.77 *Fe*
3621.46	1	.47 *Fe*

TABLE I—*Continued*

λ	Intensity	Identification
3624.17	2b	....
27.85	0	.81 *Co*
31.40	7	.36 *Co*, **.46 *Fe***
34.92	3	.72 *Co*, .95 *Ni*
37.48	1b	(*Fe*)
39.45	0	.45 *Co*
41.63	1	.65 *Ni*
42.50	1	–, **.68 *Ti***
45.06	4	.08 *Fe*
47.85	7	.84 *Fe*
49.54	0	.51 *Fe*
51.51	1	.47 *Fe*
53.48	1	.49 *Ti*
55.25	1	....
57.75	1	(*Mn, Fe, Co*)
60.27	1	.33 *Fe*
64.07	2	.10 *Ni*
66.36	4	....
69.26	5	.15 *Fe*, **.24 *Ni***
73.95	2	(*Fe, Ni*)
77.38	3	.32 *Fe*, .46 *Fe*
79.91	6	.91 *Fe*
82.60	2	....
85.45	2	.19 Ti^{+}, –
87.51	6	.46 *Fe*, .48 *V*, .66 *Fe*
89.79	0	....
93.67	1b	(*Co, Fe, Ni*)
96.26	0	....
97.59	0	.43 *Fe*, .54 *Fe*
3700.84	0	....
02.10	0	.04 *Fe*, .24 *Fe*
04.12	3	.06 *Co*
05.57	4	.56 *Fe*
07.77	3	.82 *Fe*
09.26	4	.25 *Fe*
15.57	0	....
19.90	6	.93 *Fe*
21.95	1	(*Fe*)
22.35	4	–, .50 *Ni*
24.68	2	(***Fe***, *Ti, Ni*)
27.62	7	.62 *Fe*
30.62	4	(*Fe, Co, Ni*)
33.45	1	.32 *Fe*, .49 *Co*
34.82	6	.86 *Fe*
37.13	4	.13 *Fe*
39.34	1	(***Ni***, *Fe*)
41.54	1	–, .64 Ti^{+}
43.44	3	.37 *Fe*, .48 *Fe*
45.76	5	.56 *Fe*, .90 *Fe*
48.31	7	.26 *Fe*
49.36	4	.48 *Fe*
52.97	0	....
53.14	0	.14 *Fe*
3758.28	7	.24 *Fe*

TABLE I—*Continued*

λ	Intensity	Identification
3761.03	0	.05 *Fe*
63.87	5	.79 *Fe*
67.20	5	.19 *Fe*
70.63	0	....
74.98	0	.83 *Fe*
75.80	0	.86 *Fe*
76.19	1	....
78.20	1	.06 *Ni*, .33 *Fe*
81.62	1	....
82.59	0	.45 *Fe*, .61 *Fe*
83.86	1	**.53 *Ni*,** –
86.17	3	.04 *Ti*, **.17 *Fe***
87.88	2	.89 *Fe*
88.10	3	....
89.93	3	–,**0.09 *Fe***
92.63	0	.35 *Ni*
95.02	2	.00 *Fe*
98.82	3b	.52 *Fe*
3801.79	2	.68 *Fe*, .81 *Fe*
07.09	5	6.86 *Mn*, **7.15 *Ni***
13.38	1	**2.96 *Fe*,** 3.89 *Fe*
15.90	4	.84 *Fe*
20.38	6	.43 *Fe*
25.40	4b	4.44 *Fe*, .88 *Fe*
29.23	0	.36 *Mg*
32.50	1	.31 *Mg*
33.46	4	.32 *Fe*, –
34.10	1	–, **.22 *Fe***
38.33	3	.29 *Mg*
40.68	3	.44 *Fe*, 1.05 *Fe*
43.38	0	**.26 *Fe*,** .69 *Co*
45.44	0	.48 *Co*
49.97	5	.97 *Fe*
53.47	0	.48 *Fe*
56.46	4	.37 *Fe*
58.94	2	.30 *Ni*, –
60.00	6b	9.91 *Fe*
65.68	2	.52 *Fe*
72.50	4	.50 *Fe*
78.35	5	.02 *Fe*, .57 *Fe*
82.10	5	possibly *CN* (0, 0)
86.36	5	.28 *Fe*
88.75	1	.52 *Fe*, .83 *Fe*
89.75	1	....
91.91	0	.93 *Fe*
95.57	1	.45 *Fe*, .66 *Fe*
97.80	1	.89 *Fe*
99.70	2	.71 *Fe*
3902.63	2	.26 *V*, .95 *Fe*
03.24	4	....
05.78	3	.52 *Si*, –
06.37	4	.29 *Co*, .48 *Fe*
20.70	1	**.26 *Fe*, –**
3921.74	3	....

TABLE I—*Continued*

λ	Intensity	Identification
3922.67	1	.43 *V*, .76 *Co*, **.91 *Fe***
25.58	1	....
27.93	1	.92 *Fe*
30.32	1	.30 *Fe*
32.21	0	....
34.49	0	....
41.09	0	0.89 *Co*
43.92	2	....
44.02	3	.02 *Al*
44.55	4	....
48.72	2	.68 *Ti*, .78 *Fe*
52.57	2	.61 *Fe*
56.47	0	.34 *Ti*, .46 *Fe*, .68 *Fe*
3961.55	3	.54 *Al*

uncertainty may exist as to the assignments even though the lower energy levels of the atom are quartets and many transitions are available for checking suspected lines.

Chromium has a number of strong lines arising from the multiplet a^7S-y^7P, where the (a^7S) term is the normal state. The only other important lower term is (a^5D). Transitions involving this state, however, give lines near λ 3000, a region just beyond present observations. The two strong resonance lines of *copper*, 4^2S-4^2P, were observed. One of these appears to be unblended and might be an interesting line to be studied more closely for classification purposes.

Iron, of course, shows a very large number of lines. Lines of ten multiplets were identified with a high degree of certainty. In the three multiplets a^5D-z^5D, a^5F-y^5F, and a^5D-z^5F, lines corresponding to all of the permitted transitions were observed. These multiplets in particular should be important for intensity investigations.

The assignment of *germanium* to λ 3269.50, a^1D-z^3P, may be regarded as rather doubtful, since there is little supporting evidence. *Magnesium* is characterized by the two multiplets 3^3P-5^3S and 3^3P-3^3D. All of the individual lines were observed in the first of these multiplets. No lines of *ionized magnesium* were observed. Very few lines were attributable to *manganese*, and in nearly all cases there is considerable blending.

Sodium is clearly identified by the pair of lines, 3^2S-4^2P, at λ 3302, which form the second pair of the principal series. It seems worth while to investigate this pair of lines closely in connection with the problem of interstellar sodium. Their general sharpness gives them increased value in this connection. A lack of symmetry of these lines, which is difficult of explanation, has been observed in several instances.

Nickel, like iron, shows a great number of lines. Six multiplets were observed. The multiplet a^3D-z^3F is characterized by a number of strong lines. The multiplet $a^3D-z^3D^0$ would be especially useful for intensity work, since its lines exhibit a wide range in intensity. The multiplets a^3D-y^3F and $a^3D-y^3D^0$ have most of their lines near λ 3000 and consequently were not observable. These multiplets, together with those of chromium mentioned above, probably account in large measure for the decrease in intensity of the general spectrum between λ 3000 and λ 3100. The presence of these strong resonance lines will make the detection of the major band (0, 0) of the *OH* molecule at λ 3064 an extremely difficult problem.

Silicon and *strontium* are not in evidence in α Boötis. No lines of the latter element were observed, and the one possibility for silicon, λ 3905.7, is very likely blended.

Titanium is present in abundance both in the ionized and neutral states. Seven multiplets belonging to ionized titanium and five for the neutral atom were observed.

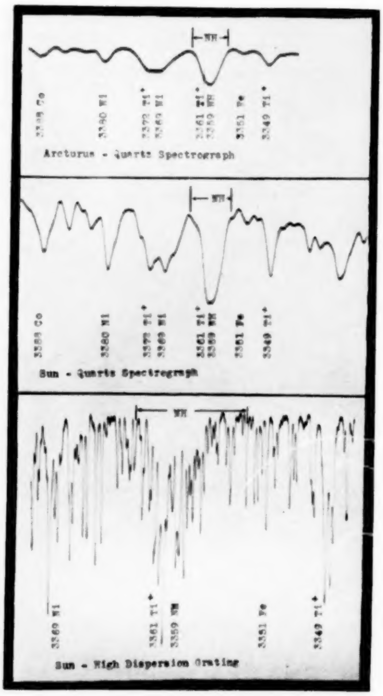
Vanadium was definitely identified by lines belonging to the multiplet a^4F-x^4G. The other expected multiplet, a^4F-x^4F, has lines near λ 3000 and probably contributes to the general decrease in intensity observed in that region. Only a few traces of *ionized vanadium* were noted. No evidence for *yttrium* and *zinc* was found.

THE *NH* MOLECULAR SPECTRUM

Perhaps the most interesting part of the spectrum of α Boötis is the molecular absorption due to the diatom *NH*. This molecular spectrum was first identified in the sun by Fowler.[4] Since it lies at λ 3360 and is to be expected only in the cooler stars, it was naturally difficult to observe it in stars other than the sun before the advent

[4] Fowler, *Phil. Trans. Roy. Soc.*, A, **218**, 351, 1919.

PLATE IX



THE SPECTRUM OF ARCTURUS

of aluminized mirrors. The absorption[5] arises from the vibrational change (0, 0), which has the usual three branches (P, Q, R), each of which in this case is triple, since the electronic change is $^3\Pi \rightarrow {}^3\Sigma$. Thus, a large number of lines lie within a very small range of wave-lengths. In the case of the sun the individual lines of the band are observable with high dispersion, but with an ordinary spectrograph all that can be expected is a single broad absorption band, more or less unsymmetrical.

The upper curve in Plate IX shows a microphotometer tracing of the spectrum of α Boötis in the neighborhood of the *NH* band. The absorption designated as *NH* appears to be composed of two strongly overlapping absorptions, as seen on the actual spectrogram. The rise in the intensity of the background in the center of this absorption is so slight, however, that it is not registered by the microphotometer. The middle curve of Figure 1 is a similar tracing of the solar spectrum photographed with the same spectrograph. The difference in detail is due to the fact that the solar spectrogram has a width several times that of α Boötis. In the solar case a trace of the splitting of the *NH* absorption into two parts is observable. This division of the absorption band at λ 3360 arises from the peculiar distribution of lines in the branches of the *NH* band, as well as the superposition of a fairly strong atomic line, Ti^+, λ 3361.

An intensive search was made to find some atomic absorber which would account for the strong absorption at λ 3360. While a few possibilities (listed in Table I) exist, all save one belong to multiplets the other members of which indicate an intensity far too small to account for the observed absorption. For example, the nickel line λ 3361.57 is one of the weakest lines of the multiplet $a^3D - z^3P^o$, for which the stronger lines were observed. Again, the line λ 3361.85 of *Ti* does not seem adequate, since only one other line—a weak one—of the multiplet $a^3F - x^5D^o$ was found. The one exception is Ti^+, λ 3361.21. This line is one of the four strong members of the multiplet $a^4F - z^4G^o$. Two other members are near by and are indicated on the tracings as Ti^+, λ 3372 and Ti^+, λ 3349. In the case of λ 3349, at least, there is no blending, and the tracings (both for α Boötis and

[5] No attempt will be made in this paper to demonstrate the presence of the (1, 1) band at λ 3370, since this is a problem in heterochromatic photometry.

the sun) give a good idea of the relative intensities. From these considerations it is clear that the total absorption at λ 3360 cannot be due to atomic lines alone.

If it is granted that the only seriously blending line is Ti^+, λ 3361, it is desirable to obtain a more accurate idea of the relative importance of the atomic absorption. The region near λ 3360 was examined with the aid of a high-dispersion spectrogram of the sun photographed by A. S. King of the Mount Wilson Observatory, in the third order of a 15-foot grating. The dispersion was about 1.25 A per millimeter. The lower tracing of Figure 1 is from this spectrogram. The tracing has been compressed somewhat by slow microphotometer drum rotation, so that it is more or less comparable with the two upper tracings. In the high-dispersion spectrogram the individual lines of the *NH* band are easily seen. Here it is clear that the absorption observed at λ 3360 is only moderately affected by the Ti^+ line. As far as spectrograms of a single star are concerned, it seems evident that the *NH* band is readily observable.

Russell[6] has shown from theoretical considerations that the abundance (log S) of *NH* molecules per unit area in both giant and dwarf stars does not vary greatly over a change in spectral class from F to M. In both instances a maximum in the abundance of *NH* is predicted near the first half of class M. A star such as Betelgeuse should thus show a slightly stronger *NH* absorption than α Boötis. In the case of very cool bodies (planets as the limiting case), the abundance of *NH* should be disturbed by the formation of ammonia. The small variation in *NH* abundance for a spectral class is of importance, however. It is known from laboratory experiments[7] that the intensity distribution of the lines of the *NH* band at λ 3360 is sensitive to temperature. Hence the study of the position of maximum absorption of *NH* should lead to interesting results regarding the variation of stellar temperature with spectral class. An extensive investigation of the *NH* band, both in stellar and laboratory sources, is now in progress.

[6] H. N. Russell, *Ap. J.*, **79**, 281, 1934.

[7] Hulthen and Nakamura, *Nature*, **119**, 235, 1929.

The author wishes to express his appreciation to Dr. A. S. King for the beautiful high-dispersion solar spectrogram used in this investigation, as well as in work being done on the *OH* diatom.[8]

DEPARTMENTS OF PHYSICS AND ASTRONOMY
CORNELL UNIVERSITY
December 10, 1935

[8] R. W. Shaw, *Ap. J.*, **76**, 202, 1932. A search, unsuccessful as yet, has been made for the *OH* band in stellar spectra. The head of the principal band of the *OH* system lies at λ 3064, or very near the Huggins ozone band region. This fact, together with the very great prominence of the *Ni* multiplets, mentioned above, and the general decrease in intensity of the ultra-violet region for late-type stars, seriously complicates the identification of the *OH* band.

A NOTE ON THE FORMATION OF STELLAR ABSORPTION LINES

P. SWINGS AND OTTO STRUVE

ABSTRACT

Microphotometer measures of the total absorptions of lines of *O* II within the wing of $H\gamma$, and of *CaH* within the wing of $H\epsilon$, show departures from the normal absorptions predicted from the multiplet intensities of these lines. The departures increase with the wing absorption, but the effect is more pronounced for *O* II than for *CaH*. Eddington's theory of absorption lines leads to a theoretical evaluation of the effect. The remaining discrepancies between this theory and actual observation are unexplained.

1. In a paper by Unsöld, Struve, and Elvey[1] attention was called to the fact that the total absorptions of stellar lines superposed over the wings of broad hydrogen lines are appreciably reduced by the effect of the hydrogen absorption, which prevents us from seeing as deep into the star's atmosphere as we would if there were no hydrogen wings. As a consequence, the ratio K/H in α Lyrae was found to be 1.92 in place of the usual value of $\sqrt{2} = 1.41$. A similar effect is also observed for the *O* II lines λλ 4346, 4347, 4349, and 4351, which fall upon the wing of $H\gamma$ in the early B-type stars. The departures of the total intensities from their normal values depend upon the amount of hydrogen absorption. The latter produces, in a sense, an additional amount of general opacity in the star's atmosphere, and it seems worth while to compare the measured line absorptions with those given by the theory.

2. The observational material consists of single-prism spectrograms, made on the fine-grain "Eastman Process" emulsion, of the following stars: κ Orionis, B0 (1 plate); ζ Persei, B1 (2 plates); τ Scorpii, B0 (3 plates); θ Ophiuchi, B2 (1 plate); α Cygni, A2 (1 plate); η Leonis, A0 (3 plates); α Lyrae, A0 (2 plates); α Canis Majoris, A0 (1 plate). The results of the measurements are given in Tables I and II. In Table III the total absorptions of the *O* II lines are reduced to an arbitrary value of 10.0 for λ 4317. Figure 1 shows an average curve of growth for those *O* II lines which are not affected by hydrogen wings. The precision of the measures is not

[1] *Zs. f. Ap.*, 1, 324, 1930.

TABLE I

B-Type Stars

Multiplet	Line	Theor. Int.	κ Orionis Abs.	κ Orionis H-Wing	ζ Persei Abs.	ζ Persei H-Wing	τ Scorpii Abs.	τ Scorpii H-Wing	θ Ophiuchi Abs.	θ Ophiuchi H-Wing
			A	per cent	A	per cent	A	per cent	A	per cent
O II $y^4P - e^4P^o$...	4317.16	39.7	0.05	0	0.14	0	0.07	0	0.05	0
	19.65	42.9	.06	0	.13	0	.08	0	.05	0
	25.77	7.9	.01	0	.05	0	.04	0	.02	0
	45.57	39.7	.07	0	.09	7	.05	12	.01	17
	49.43	100.0	.12	0	.26	3	.09	5	.03	10
	66.91	42.9	.06	0	.14	0	.08	0	.05	0
O II $y^2D - f^2D^o$...	47.43	64.3	.05	0	.06	4	.03	6	.01	12
	51.27	100.0	0.06	0	0.10	2	0.06	3	0.02	7

TABLE II

A-Type Stars

Line	Theor Int.	$\sqrt{\text{Th. Int.}}$	α Cygni Abs.	α Cygni H-Wing	η Leonis Abs.	η Leonis H-Wing	α Lyrae Abs.	α Lyrae H-Wing	α Can. Maj. Abs.	α Can. Maj. H-Wing
			A	per cent	A	per cent	A	per cent	A	per cent
H.......	1	1.	0.71	25	0.25	43	0.26	74	0.22	62
K.......	2	1.41	1.02	0	0.43	0	0.56	0	0.58	0
Ratio K/H..	......	1.41	1.44	......	1.72	......	2.15	......	2.64	......

TABLE III

Line	$\sqrt{\text{Th. Int.}}$	Outside of Wing (Mean Obs. Int. Reduced to λ 4317=10)	Inside of Wing: ζ Persei O II	ζ Persei *H*	τ Scorpii O II	τ Scorpii *H*	θ Ophiuchi O II	θ Ophiuchi *H*
4317...	6.3	10.0	......	......	......	......	......	......
20...	6.5	10.5	......	......	......	......	......	......
26...	2.8	4.0	......	......	......	......	......	......
67...	6.5	10.8	......	......	......	......	......	......
46...	6.3	7.0	6	7	7	12	2	17
49...	10.0	25.0	......	......	9	5	6	10

sufficient to construct separate curves of growth for each star. The total absorptions of lines affected by hydrogen wings are shown by crosses. They were, of course, measured from the wing of the hydrogen lines and not from the continuous spectrum.

Figure 2 shows the results of H and K for *Ca* II. Here the material is insufficient for the construction of a curve of growth. Consequently, we adopt the relation $A \propto \sqrt{\text{Th. Int.}}$

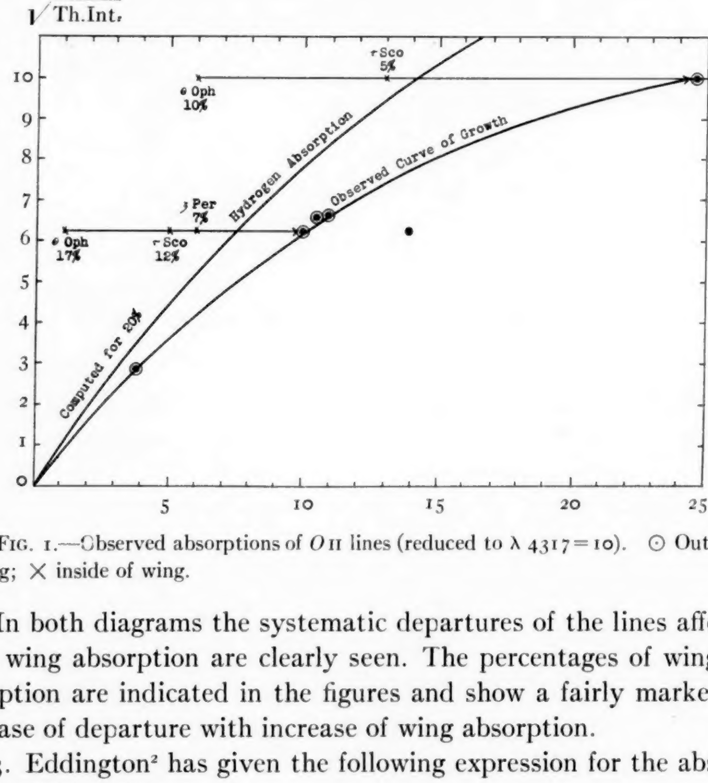


FIG. 1.—Observed absorptions of *O* II lines (reduced to λ 4317 = 10). ⊙ Outside of wing; × inside of wing.

In both diagrams the systematic departures of the lines affected by wing absorption are clearly seen. The percentages of wing absorption are indicated in the figures and show a fairly marked increase of departure with increase of wing absorption.

3. Eddington[2] has given the following expression for the absorption within a line:

$$A = 1 - \frac{1 + \frac{2}{3}q}{1 + \eta + \frac{2}{3}q},$$

where $q^2 = 3(1+\eta)(1+\epsilon\eta)$ and $\eta = l/k$ is the ratio of the absorption coefficient in the line to the coefficient of continuous absorption,

[2] *M.N.*, **89**, 620, 1929.

while ϵ is the fraction of the absorbed radiation that is lost by superelastic collisions.

We shall assume that $\epsilon = 0$; and we shall, furthermore, distinguish between the coefficients of line absorption l_0, for the oxygen

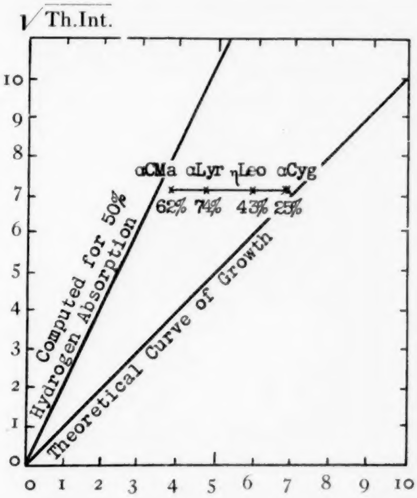


FIG. 2.—Observed absorptions of Ca II (reduced to $K = 10$)

lines, and l_H, for $H\gamma$. Then, assuming that the hydrogen wing absorption is similar to general absorption within the star's atmosphere, and that there is no stratification, we have

$$A = 1 - \frac{1 + 1.16\sqrt{1 + \frac{l_0}{k + l_H}}}{1 + \frac{l_0}{k + l_H} + 1.16\sqrt{1 + \frac{l_0}{k + l_H}}} . \qquad (1)$$

The coefficient of line absorption for O II may be written

$$l_0 = a_m f(\lambda - \lambda_0) ,$$

where a_m is a coefficient numerically equal to the theoretical multiplet intensity of the line in question. Equation (1) contains the term $l_0/(k + l_H)$. This may be written:

$$\frac{l_0}{k + l_H} = \left(\frac{a_m f}{1 + l_H/k}\right) \frac{l_0}{k} .$$

Accordingly, the same absorption A will be observed for a line in the wing as for another line outside the wing, provided that

$$a_{m'}=\frac{a_m}{1+l_H/k}\ .$$

From Eddington's paper[3] we find that for an intensity of 0.8 within the hydrogen wing the corresponding value of $\eta=l_H/k=0.62$. Accordingly, $a_{m'}=0.62a_m$. With this value of a_m we have computed the theoretical curve of growth that would correspond to a wing absorption of 20 per cent. The observed points in Figure 1 seem to indicate that the real departures are larger than those computed.

For the H line of Ca II a similar computation has been made, assuming that the wing absorption of $H\epsilon$ near the line CaH is 50 per cent. Eddington's data give for $A=0.5$, $\eta=3.43$, and consequently $a_{m'}=0.26a_m$. Figure 2 shows that the observed points have smaller departures than would be expected from the theory.

4. Similar results are obtained if, instead of assuming that the hydrogen wing absorption is similar to general absorption, we suppose that the absorption in the wing is formed by pure scattering. Let us consider first the case of an O II line appearing in the wing of $H\gamma$. The flux inside of the O II line is

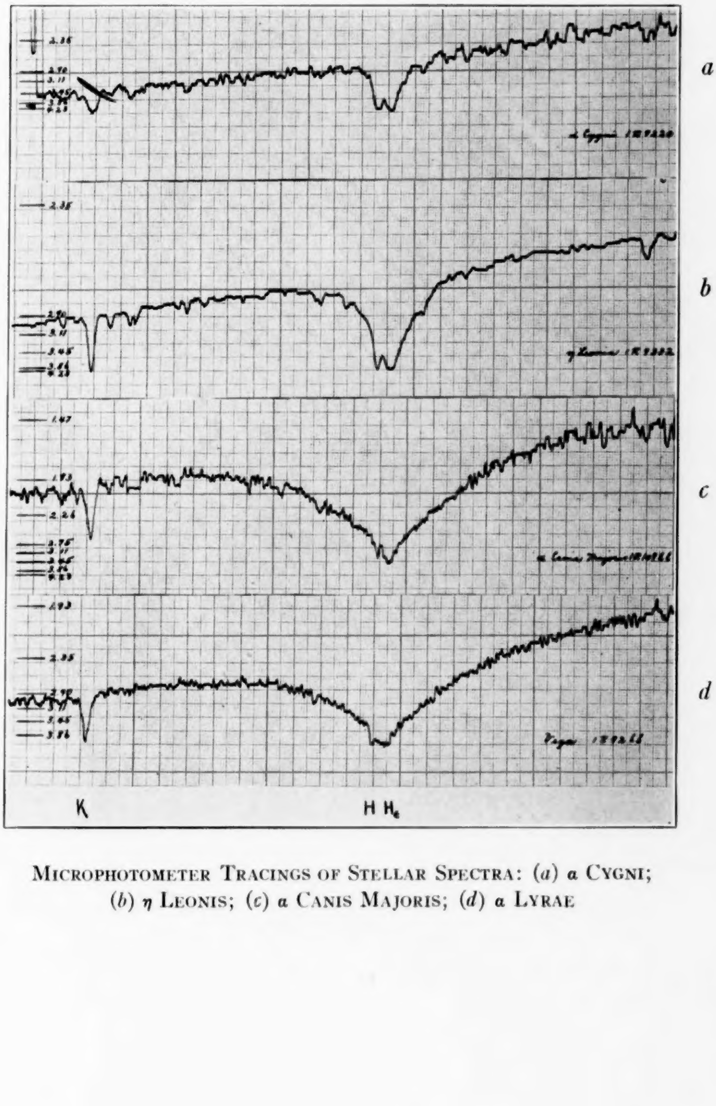
$$F\left(\frac{l_0+l_H}{k}\right)=\frac{1+1.16\sqrt{1+\dfrac{l_0+l_H}{k}}}{1+\dfrac{l_0+l_H}{k}+1.16\sqrt{1+\dfrac{l_0+l_H}{k}}}\ ;$$

the flux outside of the O II line is

$$F\left(\frac{l_H}{k}\right)=\frac{1+1.16\sqrt{1+\dfrac{l_H}{k}}}{1+\dfrac{l_H}{k}+1.16\sqrt{1+\dfrac{l_H}{k}}}\ .$$

[3] *Loc. cit.*

PLATE X



MICROPHOTOMETER TRACINGS OF STELLAR SPECTRA: (*a*) α CYGNI; (*b*) η LEONIS; (*c*) α CANIS MAJORIS; (*d*) α LYRAE

Thus the absorption in an O II line appearing in the wing of $H\gamma$ is

$$A = 1 - \frac{F\left(\frac{l_0 + l_H}{k}\right)}{F\left(\frac{l_H}{k}\right)} = 1 - \frac{\left(1 + 1.16\sqrt{1 + \frac{l_0 + l_H}{k}}\right)\left(1 + \frac{l_H}{k} + 1.16\sqrt{1 + \frac{l_H}{k}}\right)}{\left(1 + \frac{l_0 + l_H}{k} + 1.16\sqrt{1 + \frac{l_0 + l_H}{k}}\right)\left(1 + 1.16\sqrt{1 + \frac{l_H}{k}}\right)}.$$

For l_H/k we may use Eddington's table; for l_0/k we use the same table, considering the percentage of absorption that the O II line would have outside of the wing.

The results of the calculations for the two lines 4346 and 4349 are summarized in Table IV; the third column for each star gives the reduction factors for the intensities of the lines appearing in the wing. It is obvious that these factors are much smaller than the observed values.

TABLE IV

COMPUTED REDUCTIONS OF THE INTENSITIES OF THE O II LINES

Line	ζ Persei			τ Scorpii			θ Ophiuchi		
	l_H/k	l_0/k	Reduction Factor	l_H/k	l_0/k	Reduction Factor	l_H/k	l_0/k	Reduction Factor
4345.57	0.2	0.4	0.92	0.36	0.26	0.9	0.5	0.2	0.81
49.43	0.08	0.66	0.95	0.15	0.42	0.93	0.3	0.35	0.86

The opposite conclusion is reached for the H line of Ca II, as is shown in Table V. The computed ratios $\frac{\text{Intensity of K}}{\text{Intensity of H}}$ are increasing in the same direction as the observed values, but are much too large.

5. The opposite sense in which the two sets of observations depart from the theory is rather puzzling. Our assumptions for l_H may be erroneous, but it does not seem likely that this is the cause of the discrepancy. One might be tempted to interpret the results as an effect of stratification.[4] But it is quite improbable that in the B

[4] This effect of stratification must be carefully defined. In considering the scattering hypothesis, we assume that all the atoms are completely mixed; but the lines of O

stars the hydrogen wings are formed above the *O* II lines while in the A stars they are formed below the *Ca* II lines. The observations are not very accurate and large systematic errors may, conceivably, have affected our results for *O* II. Nevertheless, the departure from the normal intensities of λλ 4346, 4349, 4347, 4351 is apparent even from a visual inspection of the spectrograms, while the computed departures for wing absorptions of the order of 10 per cent should hardly be noticeable. Under the circumstances it seems best to await more accurate observational data before an attempt is made to explain the phenomenon.

TABLE V

COMPUTED RATIOS OF THE INTENSITIES OF THE K AND H LINES OF *Ca* II

Star	$\frac{l_{H\epsilon}}{k}$	$\frac{l_{Ca\,\text{II}}}{k}$	Computed Ratios K/H	Observed Ratios K/H
α Cygni	0.85	3.8	1.82	1.44
η Leonis	1.6	1.15	2.62	1.72
α Lyrae	16.	2.15	5.8	2.15
α Can Maj	7.	1.45	6.	2.61

In conclusion, attention may be called to the fact that the stellar intensities of blended lines present a problem that is essentially similar to that discussed here. Evidently it is not permissible in radial-velocity work to take simply the weighted mean for the wave-length of the blend, using for the weights the laboratory intensities of the lines. Whenever the intensities of the components are not the same, the blended wave-length will depend upon the shapes of the two lines. For example, the wave-length of $H\epsilon$ should be much less affected by *Ca* II *H* in Sirius than in η Leonis.

We wish to express our thanks to Dr. Chandrasekhar for several valuable suggestions.

YERKES OBSERVATORY
November 24, 1935

and *H* (or of *Ca* II and *H*) originate in different levels; this appears clearly when the flux inside of a line is plotted as a function of the depth. On the other hand, we may have a real chemical stratification of the elements. It is this type of stratification that we are interested in.

ON THE SPECTRUM OF THE SUPERNOVA S ANDROMEDAE

CECILIA PAYNE GAPOSCHKIN

ABSTRACT

According to the synopsis of contemporary spectroscopic observations given in the present note, the spectrum of S Andromedae contained bright lines, which presented little contrast to the background even when the continuous spectrum had grown comparatively faint. It is concluded that the spectra of supernovae (so far as the existing observations go) are not essentially different from those of more ordinary novae, although the details are very different, and far larger velocities are probably involved.

There appears to be a general opinion that the evidence concerning the spectrum of the most luminous nova of modern times was so contradictory that conclusions as to its spectral nature are impossible. This view is expressed, for example, by Miss Cannon: "With the testimony apparently so conflicting, it is difficult to form any conception of the class of this spectrum. It may have been similar to that of Nova Centauri, which also appeared in a nebula, and which resembled Class R."[1]

As described in a preceding note, it now appears that Nova Centauri had a gaseous spectrum differing in degree rather than in kind from those of other novae. It is therefore of some interest to examine the records of the spectrum of S Andromedae, which was a sixth-magnitude star, and was observed by experienced spectroscopists, using the best equipment at their disposal. When this is done, it appears, in the writer's opinion, that while some of the observed differences were colored by point of view, much of the discordance is removed by arranging the observations in chronological order. It must be remembered that the normal development of the spectrum of a nova was less well known in 1885 than at present, and that observations of a rapidly changing spectrum might well have appeared discordant. This point is clearly brought out by the fact that several observers published spectroscopic observations without noting the exact dates; they were evidently unaware that a considerable change was possible. It is perhaps not generally

[1] *Harvard Ann.*, **76**, 35, 1917.

TABLE I

OBSERVATIONS OF THE SPECTRUM OF S ANDROMEDAE

Date, 1885	Observer	Reference	Observation	Note
Sept. 1	Young	Sid. Mess., **4**, 282, 1885	No lines seen.	
Sept. 1	Copeland	*M.N.*, **47**, 49, 1886	Continuous from end to end, and only on close examination could slight condensations indicative of bright lines be detected.	
Sept. 1, 2	Vogel	*A.N.*, **112**, 283, 1885	Spectrum continuous, red and yellow strong; green weak. A hazy dark band between F and G.	1
Sept. 1, 2, 3	Cacciatore	*A.N.*, **112**, 300, 1885	"Spectre du noyau subtile et linéaire, avec indice incertain de nœuds plus luisants."	
Sept. 5 or before	Copeland	*A.N.*, **112**, 286, 1885	Fairly continuous spectrum.	
Sept. 3	Lohse	*M.N.*, **46**, 300, 1885	Continuous; no lines seen.	
Sept. 3	Huggins	*Observatory*, **8**, 333, 1885	Low-dispersion spectrum seen from C to beyond F; apparent condensation of light from D to b, which might be due to bright lines. Suspicion strengthened by a more powerful spectroscope, but not able to be certain.	
Sept. 4	Maunder	*Observatory*, **8**, 335, 1885	Continuous spectrum; no lines seen, bright or dark.	
Sept. 4 or later	Konkoly	*A.N.*, **112**, 286, 1885	"Es macht sicher den Eindruck, als sähe man helle Felder auf dunklem Grunde, und zwar sieht man im Roth, Gelb, Grün, und Blau solch eine breite Bande. Wenn die Sache so wäre, so würden diese breiten hellen Felder den H-linien C and F, sowie D_3 entsprechen, *und zwar einem enorm hohen Druck.* Allerdings sieht man auch noch ein solches breites Feld im Grün, welches sicherlich nicht zur genannten Gruppe gehören kann. *Ich wäre eher geneigt, das Spectrum zum Typus IIIb zu rechnen, und damit wäre die Theorie des Herrn Prof. Ritter in Aachen wohl bestätigt.* Es ist nicht zu übersehen, dass im Spektrum der violette Theil förmlig fehlt . . . ." (italics ours).	2

TABLE I—*Continued*

Date, 1885	Observer	Reference	Observation	Note
Early Sept.	Ricco	*Nature*, **32**, 523, 1885	Spectrum continuous, with suspected bright bands.	
Sept. 5	Sherman	*Amer. Sci.*, Nov., 1885	Bright lines at 5575, 5315, 4861.	3
Sept. 6	v. Gothard	*A.N.*, **112**, 390, 1885	Continuous spectrum; certainly no dark lines; no bright lines seen, but might easily have been missed.	
Sept. 7	Lord Rosse	*Nature*, **32**, 437, 1885	Star reddish-yellow, like Aldebaran; continuous spectrum, with bright band or line in the green.	
Sept. 9	Lohse	*M.N.*, **46**, 301, 1886	Spectrum continuous; part at G appears brighter.	
Sept. 9	Huggins	*Observatory*, **8**, 334, 1885	"I was so far confirmed in my suspicion of bright lines, that I have little doubt that from three to five bright lines were present between D and b."	
Sept. 10	Vogel	*A.N.*, **112**, 302, 1885	Spectrum continuous, without special peculiarities. "Bei den Beobachtungen in den ersten Tagen von September hatte ich einigemale den Eindruck, als ob im Roth und besonders im Gelb helle linien im Spektrum vorhanden seien."	
Sept. 11	Maunder	*Observatory*, **8**, 335, 1885	Nova spectrum shorter at both ends than that of an ordinary star. Bright lines suspected at 5480, 5327; no bright lines at D, F, or b.	4
Sept. 13	Perry	*M.N.*, **46**, 22, 1884	Bright band in green suspected.	
Sept. 15	Maunder	*M.N.*, **46**, 20, 1885	Two bright lines, as on Sept. 11, suspected.	
Sept. 16	Backhouse	*M.N.*, **48**, 110, 1888	"Its spectrum is partly continuous, but there is certainly interrupted light as well, which must be either bright lines or short bright spaces between broad absorption bands; I believe the former. There are certainly two bright spots . . . . , one probably in the green, much plainer than the other which is halfway from it to the red end. I strongly suspect a third at an equal distance on the other side of the brightest."	

TABLE I—*Continued*

Date, 1885	Observer	Reference	Observation	Note
Sept. 18–20	Vogel	*A.N.*, **112**, 387, 1885	Spectrum continuous. "Nach meinen Beobachtungen gehört das Spektrum *nicht* zu Classe III*b*, denn die Banden im Gelb und Blau, die ich bei den ersten Beobachtungen, wo der Stern hell war, wahrgenommen habe, waren nicht sehr breit."	
Week before Sept. 29	Seabroke	*Nature*, **32**, 523, 1885	A bright line near to, and violet, of D.	
Sept. 30	Maunder	*M.N.*, **46**, 20, 1885	Two bright lines, as before, better seen, though the star was fainter. A third line, at 5575, suspected.	4
Nov. 5	Backhouse	*M.N.*, **48**, 110, 1888	"There can be no doubt that the spectrum of the new star is highly interrupted . . . . contains more than one definite bright line."	

REMARKS TO TABLE I

1. The line designations are those of Fraunhofer: C = $H\alpha$; D = 5890 (*Na*); D3 = 5875 (*He*); b = *Mg* lines at 5170; F = $H\beta$; G = 4315 (C_nH_n).

2. This description is given in full because it seems to be the sole basis for the statements that some observers saw a fluted spectrum. It will be noted that Konkoly describes the spectrum as consisting of bright bands, and then states his conclusion that it is of class III, or an M star in the modern notation. His immediate statement that such a spectrum would substantiate the Ritter theory, his mention of "enormously high pressure," and his remarks on the "band" in the green lead one to suspect that his classification was not free from bias, and apart from his expression of opinion there is nothing in his description to conflict with those of other observers.

3. The bright lines measured by Sherman in various spectra were questioned by Vogel (*A.N.*, **113**, 389, 1886); it may be noted, however, that two of the lines he measured for S Andromedae coincide rather closely with lines measured by Maunder (4).

known that Nova Persei (1901) was the first nova for which a pre-maximum spectrum, consisting chiefly of absorption lines, was observed. It is still remembered at Harvard that when the earliest spectrum of Nova Persei was developed, the observers did not at first believe that the right star had been photographed.

The principal observations of the spectrum of S Andromedae are summarized in Table I. The actual words are not given, unless they are inclosed in quotation marks.

A survey of Table I seems to lead to the conclusion that the evidence is consistent with a nova spectrum that was at first practically continuous, and later showed bright lines of no very great intensity.[2] The "continuous" stage seems to have occurred near maximum, which took place, according to Maunder, on August 31 or September 1, 1885.[3] This was also the opinion of many astronomers at the time, as may be seen from the summary of the spectroscopic observations given by Maunder:

> It seems probable . . . . that the star has a double spectrum, but that . . . . the relative intensity of [the two spectra] has suffered alteration, and that the continuous spectrum is now fainter than at first, and therefore the bright lines are more readily seen. . . . . The explanation of [the] discrepancies is to be found in the fact that the irregularities in the spectrum are slight and that the restriction placed upon the use of a wide slit or of a cylindrical lens, through the star being seen on so bright a background, render their detection exceedingly difficult and uncertain.[4]

It would seem to be unnecessary to add any comment to this one, made with full competence at the time of the observations; but the idea seems to persist that the nature of the spectrum was uncertain. Be it remarked that Vogel, Huggins, and Lord Rosse were observers of the first rank, and that on their evidence alone the existence of a bright-line spectrum seems to be beyond doubt.

[2] The description by W. W. Campbell of the spectrum of Z Centauri, which seems beyond reasonable doubt to have been a bright-line spectrum, recalls some of the observations in Table I.

[3] Note, however, the earlier maximum shown by the light-curve given by Lundmark (*Kungl. Svensk. Vetens. Hand.*, **60**, No. 8, 55, 1917), based on the recollections, possibly exaggerated, of those who, after the discovery had been announced, remembered that they had seen a star in the position of the nova.

[4] *Observatory*, **8**, 323, 1885.

It is of interest to consider what conclusions can be drawn as to the precise spectral changes involved. There is no definite note of any dark lines, and certainly the bright lines seem to have been fainter than is usually the case with post-maximum novae. In the writer's opinion, based partly on the recent interpretation of the spectrum of Nova Centauri, the lines of S Andromedae, both bright and dark, must have been enormously wide, possibly because the

TABLE II

OBSERVATIONS OF THE COLOR OF S ANDROMEDAE

Date, 1885	Observer	Observation
Aug. 31	Kiel Observatory	Slightly reddish
Sept. 1	Deichmüller	Reddish
Sept. 1	Schrader	Deep orange
Sept. 1	Engelhardt	Yellowish
Sept. 1	Millosevich	Yellow
Sept. 3	Baxendell	Light orange, nebula quite fiery
Sept. 3	Deichmüller	Reddish
Sept. 3	Hartwig	Rather reddish
Sept. 3	Baxendell	Very light orange, nebula ruddy
Sept. 4	von Speissen	Reddish
Sept. 5	Baxendell	Very light orange, nebula ruddy
Sept. 7	Baxendell	Perhaps slightly more colored
Sept. 8	Baxendell	Dull orange
Sept. 9	Baxendell	Dull yellow
Sept. 12	Baxendell	Color still orange; nebula not now much colored
Sept. 13	Baxendell	Color unchanged, but nebula not so ruddy as before
Sept. 14	Millosevich	Whiter than before

outburst of a supernova would seem to be a more violent one, and would thus be characterized by greater radial velocities, than that of a nova of more usual type. Well-known nova lines occur at λ 5325 and λ 5575; but other lines are as conspicuous, or more so, in the spectra of other novae; to make definite line identifications would be to press the material too far.

A further point of interest is to be found in the observations of the color of S Andromedae, which are summarized in Table II, with notes made by the observers.

From Table II it appears that the nova was reddish at the outset, and grew yellower as it faded, as is also stated in general terms by

Engelmann.[5] The redness of the nova may perhaps be ascribed to the strength of $H\alpha$, though the line was so widened by radial velocity that it was difficult to see. The reddening of the nebula, reported by the Baxendells, is an example of observation colored by preconceived ideas. As Dr. Fred L. Whipple has pointed out to me, such an area of reddening would take about three hundred years to attain a diameter of a minute of arc at the distance of the nebula, assuming the excitation to travel with the greatest possible velocity, that of light.

HARVARD COLLEGE OBSERVATORY
March 1936

[5] *A.N.*, **112**, 323, 1885.

NOTES

A POSSIBLE INTERPRETATION OF THE ABSORPTION SPECTRA OF NOVA HERCULIS

ABSTRACT

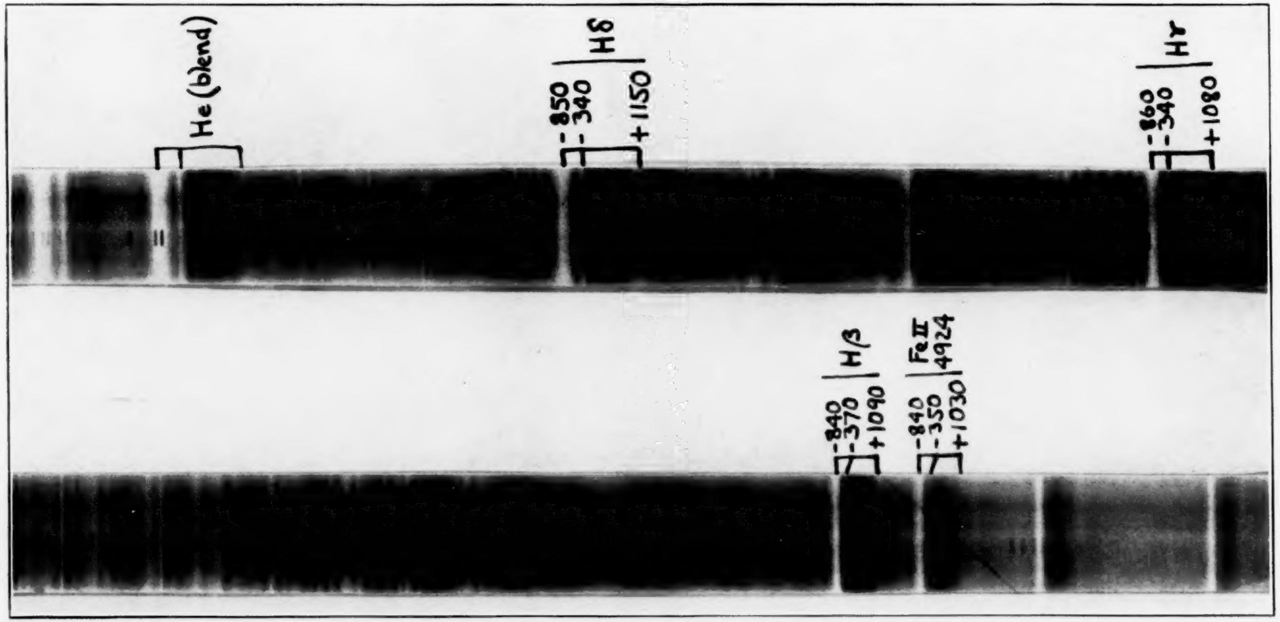
A diffuse-line absorption spectrum having a high positive radial velocity has been measured on spectrograms of Nova Herculis which had been obtained in January, 1935. The mean radial velocity measured for $H\beta$, $H\gamma$, Hδ, and *Fe* II 4924 is $+1090$ km/sec. A similar, but much stronger, absorption spectrum gives a radial velocity of -850 km/sec. It is possible that these spectra may be interpreted as arising from two starlike components ejected from the Nova similar to those observed visually by Kuiper.

The present note is a preliminary announcement of one feature of the spectrum of Nova Herculis. Spectrograms of the nova taken at the Yerkes Observatory are being measured at present and will be discussed more fully at a later time.

Some time ago absorption components having a large positive displacement were measured in the case of the Balmer lines Hβ, Hγ, Hδ, and Hϵ, and of *Fe* II 4924. Although at that time there could be no doubt of the reality of the lines measured, the general complexity of the spectrum prevented a reasonable interpretation of the lines. Also, in the case of the strongest of the lines ($H\delta$) there is a possibility that part of the observed intensity is due to a blend from a spectrum having a negative displacement. The lines of the positive spectrum were quite diffuse and were similar in appearance to a stronger set of lines having a large negative velocity.

After the discoveries of the duplicity of the nova by Kuiper and of the increasing separation of the components by Van Biesbroeck, a possible interpretation of the spectrum was suggested. Plate XI gives a reproduction of a spectrum of the nova obtained on January 27, 1935. In addition to the broad emission lines and the sharp absorption spectrum, which are interpreted as being due to an expanding gaseous shell surrounding the central star, there are two sets of absorption components: (I) a broad, strong set having a velocity of -850 km/sec; (II) a similar, but much weaker, set, at a

PLATE XI



SPECTRUM OF NOVA HERCULIS OBTAINED ON JANUARY 27, 1935

This dispersion of the original plate is 26 A per millimeter at $H\gamma$. The reproduction is a direct enlargement (not vertically enlarged). Radial velocities of the strongest lines are given for: (1) the strong diffuse absorption spectrum having a velocity of -850 km/sec; (2) the weak diffuse absorption spectrum having a velocity of $+1090$ km/sec; (3) the sharp absorption spectrum having a velocity of -350 km/sec. The range in wave-length is from λ 3915 to λ 5200. The two strong lines toward the red from λ 4925 are *Fe* II 5018 and *Fe* II 5169. The spectrogram was obtained on Eastman Hyper-press emulsion and was developed in a special fine grain developer. A comparison spectrum of *Ti* and *Fe* spark was superposed on the nova spectrum.

radial velocity of +1090 km/sec. If we assume that at the time of the outburst two large dense masses at opposite ends of a diameter of the star were expelled, it is possible to give a fairly reasonable explanation of the early spectral features. The sharp absorption spectra and the broad emission lines may be attributed to a shell of gas expanding from the central star. The strong diffuse absorption spectrum having a radial velocity of −850 km/sec may be considered as originating in the atmosphere of the brighter component which was expelled from the central star; the positive spectrum is due to the fainter of the two masses.

At the time of the rapid fading of the nova in April, 1935, double emission lines appeared having a relative velocity considerably less than that of the absorption spectra I and II. It seems very probable that these double emission lines originated in the two visual components. If absorption spectra I and II originated in the same components, a negative acceleration of the velocity and a considerable decrease in the density of the visual pair must be assumed. At the time of brightening to secondary maximum, the actual increase in light was probably due to components I and II, as the central star apparently remained faint. This would also explain why the nova was not suspected of elongation or duplicity earlier; the central star would cause the two other components to remain unresolved.

It should be emphasized that the foregoing hypothesis is advanced tentatively; there are objections to it, but they do not seem to be as unanswerable as those raised by an attempt to explain all of the observed phenomena exclusively on the assumption of the expanding-shell hypothesis.

W. W. MORGAN

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March 15, 1936

A USEFUL FINE-GRAIN DEVELOPER FOR SPECTROGRAPHIC PHOTOGRAPHY

ABSTRACT

A fine-grain developer of the physico-chemical type using metol, glycin, and paraphenylene-diamine as developing agents has been found to give stellar spectrograms on the fastest commercial emulsions of a quality almost equal to those taken on Process plates. The saving in exposure time with the fast plates is very great.

The greatly increased interest in minature camera photography during the last few years has resulted in a number of formulae designed to give fine grain on photographic plates and films. Unfortunately, most of such formulae require increased exposure times and give rather low contrast, thus making them of limited application to astronomical photography.

Recently, however, developers of the "compromise" type have appeared, which, while of high effective energy, give good contrast and grain of a size almost equal to that which can be obtained with the best of the less energetic developers. Especially useful is the paraphenylene-diamine–metol–glycin formula given below, which, when used with the fastest plates, gives contrast and grain which are little inferior to those obtained with emulsions of the Process or lantern-slide type. Experiments at the Yerkes Observatory have shown that stellar spectrograms obtained on emulsions of the type of Eastman Hyper-Press and Ilford Astra II and developed in such a formula show almost as much fine detail as those obtained on Process plates, with, of course, a great saving in the exposure time. The formula, which is given in several monographs[1] on fine-grain development is:

Paraphenylene-diamine (Edwal)	10 grams
Glycin (Edwal)	5 grams
Metol (Agfa) or Elon (Eastman)	6 grams
Sodium sulphite (Edwal fine grain) anhydrous	90 grams
Water	1 liter

Directions for mixing do not agree with those published. For the best results, the paraphenylene-diamine should be dissolved first;

[1] E.g., Wolfman, *The Fine Grain Negative* (Canton, Ohio: The Fomo Publishing Co., 1935), p. 56.

it is followed by the sulphite and then by the glycin and the metol. The development time is 12–20 minutes at 65°.

The brands of chemicals mentioned are those actually used and found to be satisfactory here. Other standard kinds probably would be satisfactory, although difficulty has been experienced with certain brands of paraphenylene-diamine.

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March 15, 1936

REVIEW

Annals of Science: A Quarterly Review of the History of Science since the Renaissance. **1**, No. 1 (January 15, 1936). Pp. 113. Figs. 18; Pls. XI. London: Taylor & Francis, 1936. Annual subscription, £1, post-free.

The editors of this new periodical are Douglas McKie (University College, London), Harcourt Brown (Washington University, St. Louis, Missouri), and H. W. Robinson (Librarian of the Royal Society, London).

> In *Annals of Science* the field of study is proposed to be restricted to the origins and growth of Modern Science, as distinct from the Mediaeval and Ancient . . . . an independent English review, dealing with the development of Modern Science, from its beginnings in the Renaissance down to its extraordinary advances in recent times, and the manner and stages of its growth and differentiation during the intervening centuries.

The restrictions imposed by this declaration of editorial policy and by the subtitle of the *Annals* receive, fortunately, a liberal interpretation. A study of the origins and growth of modern science implies, of course, a study both of its pre-Renaissance beginnings and of its industrial applications; the study of a tree limited to the trunk and foliage, to the exclusion of the roots and of the fruits, would be rather limited in scope. The paper on "Early Nautical Charts," by N. H. de Vaudrey Heathcote, on the one hand, and the first of a series of papers on "The History of the Chile Nitrate Industry," by M. B. Donald, on the other, may serve as illustrations of the futility of a strict construction of the term "History of Science since the Renaissance." The paper on "The Detection and Estimation of Electric Charges in the Eighteenth Century," by W. Cameron Walker, is of interest to historically minded students of astrophysics. The paper on "The Real Character of Bishop Wilkins," by E. N. da C. Andrade, might be quoted as an example of the bridging of the gap between science and the humanities.

A. Pogo